Math 1031
Fall 2006 Final Exam Problems

This exam contains 20 multiple-choice questions, worth 5 points each, and 8 written-answer problems, worth 12 - 14 points each, for a total of 200 points.

A reference page of useful formulas follows these problems.

**Problem 1.** Suppose you flip a fair coin 4 times, then roll a fair die 3 times.
What is the probability that this experiment results in the sequence: H - T - H - T - 1 - 2 - 3?

a) \( \frac{1}{2^4 \cdot 3^4} \)
b) \( \frac{1}{(2^4 \cdot 6^3)} \)
c) \( \frac{1}{2^4} + \frac{1}{3^3} \)
d) \( \frac{1}{4^2} + \frac{1}{3^3} \)

**Problem 2.** A state's automobile license plates consists of 3 letters followed by 3 numerals. Repeats are allowed in both the letters and the numerals, however the letters O (oh) and I (eye) cannot be used. How many different license plates are possible with this scheme?

a) \(26^3 \cdot 10^3\)
b) \(26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8\)
c) \(24^3 \cdot 10^3\)
d) \(24 \cdot 23 \cdot 22 \cdot 10 \cdot 9 \cdot 8\)

**Problem 3.** Two fair dice are rolled. What is the probability that the numbers showing on the top faces are equal or that their sum is 8?

a) \(\frac{1}{6}\)
b) \(\frac{7}{36}\)
c) \(\frac{5}{18}\)
d) \(\frac{11}{36}\)
Problem 4. Suppose you toss a fair coin 6 times. What is the probability of obtaining exactly two heads in the 6 tosses?

a) 0.3358  
b) 0.2344  
c) 0.7500  
d) 0.4850  
e) 0.3353

Problem 5. Find the equation of the line with y-intercept = 3, parallel to the line 4x + 2y = 3.

a) 2x + y = 6  
b) 2y - x = 6  
c) 2y - x = 3  
d) y = 2x + 3  
e) y = -2x + 3

Problem 6. The solution set of the inequality \( x^2 - x - 2 \geq 4 \) is:

a) \([-1, 2]\)  
b) \((-\infty, -1]\)  
c) \([-2, 4]\)  
d) \((-\infty, -2] \cup [3, \infty)\)  
e) None of the above

Problem 7. Simplify: \( \frac{2x^2 - 4x - 6}{x^2 - 4x - 5} \)

a) 2  
b) \(\frac{2(x - 3)}{x - 5}\)  
c) \(\frac{x + 3}{2(x + 5)}\)  
d) \(\frac{x + 1}{x^2 - 4x - 5}\)  
e) \(x^2 - 4x - 5\)

Problem 8. What are the x- and y- intercepts of the graph of \(4x^2 + 16y^2 = 4\)?

a) \((\pm \frac{1}{2}, 0), (0, \pm 1)\)  
b) \((\pm 1, 0), (0, \pm \frac{1}{2})\)  
c) \((\pm \frac{1}{2}, 1), (0, \pm 1)\)  
d) \((0, 1)\)  
e) None of the above
Problem 9. The inverse of the function \( f(x) = \sqrt[3]{2x + 1} \) is:

a) \( f^{-1}(x) = \frac{(x^3 - 1)}{2} \)

b) \( f^{-1}(x) = (2x + 1)^{-\frac{1}{3}} \)

c) \( f^{-1}(x) = (2x + 1)^{\frac{1}{3}} \)

d) \( f^{-1}(x) = 2x + 1 \)

e) \( f^{-1}(x) = 2(x^3 + 1) \)

Problem 10. The distance between the points (-4,-1) and (-1,3) is:

a) \( \sqrt{13} \)

b) \( \sqrt{41} \)

c) 5

d) 7

e) None of the above

Problem 11. A room is 1.5 times as long as it is wide. The perimeter is 75 feet. What are the dimensions of the room?

a) 17.5 x 20

b) 15 x 22.5

c) 18 x 27

d) 12.5 x 25

e) 15 x 25

Problem 12. Given the following equation: \( 28x - 49x^2 = 4 \) What are all the real solutions for this equation?

a) \( \frac{2}{7} \)

b) \( -\frac{2}{7} \)

c) \( -\frac{59}{98}, +\frac{3}{98} \)

d) There are no solutions

e) None of the above

Problem 13. Solve the following equation for \( x \): \( |x^2 + 6x| = 3x + 18 \)

a) -3, +3, +6

b) -3, +3, -6

c) -6, -3

d) +6, +3

e) -3, +6, -6
Problem 14. Find the domain of the following function: \( f(x) = \frac{\sqrt{x^2 - 3}}{x - 5} \)

a) \((-\infty, -3) \cup (3, \infty)\)
b) \((-\infty, -3) \cup [3, \infty)\)
c) \([3, 5) \cup (5, \infty)\)
d) \((-\infty, -3) \cup [3, 5) \cup (5, \infty)\)
e) \((-\infty, -3) \cup (3, 5) \cup (5, \infty)\)

Problem 15. The graph of the function \( f(x) = \sqrt{x} \) is transformed by reflection about the x-axis, vertical stretching by a factor of 4, vertical shifting upward up by 5 units and horizontal shifting to the left by 3 units. The resulting graph is the graph of a function \( g(x) \), where \( g(x) \) is:

a) \(-4\sqrt{x} - 3 + 5\)
b) \(-\frac{4\sqrt{x}}{3} - 5\)
c) \(+4\sqrt{x + 5} - 3\)
d) \(-4\sqrt{x + 3} + 5\)
e) \(-4\sqrt{-x + 3} + 5\)

Problem 16. A quadratic function has a vertex at \((-3,5)\) and passes thru the point \((-6,-1)\). What is the equation of this function?

a) \( y = -\frac{2}{3} (x + 3)^2 + 5\)
b) \( y = -\frac{6}{9} (x + 3)^2 + 5\)
c) \( y = -\frac{6}{9} (x + 5)^2 + 3\)
d) \( y = -2(x + 3)^2 + 5\)
e) \( y = -5(x - 5)^2 - 3\)

Problem 17. The day you were born, money was deposited in a trust fund that guaranteed paying 6.5% interest compounded continuously, for the duration of the account. How much money should have been deposited in this account if you are to receive \$500,000 on your 25th birthday?

a) \$20,000
b) \$82,258.23
c) \$39,455.84
d) \$114,099.76
e) \$98,455.84
Problem 18. Rewrite the following logarithmic expression as a natural logarithm:  \( \log_8 12 \).

a) \( \frac{\ln 12}{\ln 8} \)

b) \( \frac{\ln 8}{\ln 12} \)

c) \( \frac{\log 12}{\log 8} \)

d) \( \frac{\log 8}{\log 12} \)

e) None of the above

Problem 19. Solve the following equation for \( x \):  \( e^{2x} + 8e^x - 33 = 0 \)

a) -3, +11

b) +3, -11

c) +1.0986

d) +1.079

e) +2.890

Problem 20. Solve the following equation for \( x \):  \( \ln(5 - x) = \ln(x + 1) - \ln(x - 1) \)

a) -2, +3

b) +2, -3

c) +1, -1

d) -2, -3

e) +2, +3
Problem 21. [12 points] Let the following equation be given, \(6^{2x+3} = 81\).

a) Find an exact logarithmic solution \(x\) for this equation.

b) Find a 4-decimal approximation to your previous exact solution.

Problem 22. [12 points] An unfair coin is tossed, whose probabilities are: \(P(\text{Head}) = \frac{5}{8}\) and \(P(\text{Tail}) = \frac{3}{8}\).

If the coin toss results in a head, then a ball is drawn from an urn (called Urn \#1) containing 4 red balls and 4 white balls.

If the coin toss result is a tail, then a ball is drawn from a different urn (called Urn \#2) containing 6 red balls and 10 white balls.

(a) What is the probability that the coin toss results in a head and a red ball is drawn?

(b) What is the total probability of drawing a red ball (i.e. including the situation where the coin toss results in a head and the case where the coin toss results in a tail)?

Problem 23. [14 points] Let \(f(x) = -x^2 + 2x + 3\).

(a) Write this quadratic function in standard form.

(b) Find the vertex of the graph of \(f\) and find any \(x\)- and \(y\)-intercepts.

(c) Sketch the graph of \(f\).

(d) Find the increasing and decreasing intervals for \(f\).

Problem 24. [12 points] A gambler rolls a fair die. If she obtains a 5 or a 6, she wins. Otherwise she loses. Suppose that the gambler rolls the die 10 times in succession.

a) What is the probability that she wins exactly twice during these 10 trials? You are not required to simplify your answer, but it must be explicit (i.e. given in terms of sums, products, quotients, factorials, etc. of actual numbers).

b) What is the probability that she wins at least twice during these 10 trials? You are not required to simplify your answer, but it must be explicit.

Problem 25. [12 points] Consider the circle given by the equation

\[5x^2 + 5y^2 - 6x + 2y - 9 = 0.\]

Write the equation for this circle in standard form, and then find the center and the radius.
Problem 26. [14 points] Let \( f(x) = |x| \).

(a) Use algebra to check symmetry for the graph of \( f \) about both the \( x \)-axis and the \( y \)-axis. Your algebra should show the reasons for your answers.

(b) Apply the horizontal line test to the graph of \( f \). Based on this test, does \( f^{-1} \) exist?

(c) Let \( g(x) = 2 - 3|x - 1| \). Describe very briefly the sequence of transformations needed to obtain the graph of \( g \) from the graph of \( f \). Then sketch the graph of \( g \).

(d) What is the range of the function \( g \)?

Problem 27. [12 points]

(a) Let the function \( f(x) \) be given by \( f(x) = 2 - \frac{2}{x + 1}, \ x \neq -1 \).

Find the formula for \( f^{-1}(x) \). (For partial credit, set up the equation that gives \( f^{-1}(x) \) even if you are not able to solve it.)

(b) Verify that your solution \( f^{-1} \) for part (a) really is the inverse of the function \( f \). To do this you are expected to find \( (f^{-1} \circ f)(x) \) and \( (f \circ f^{-1})(x) \), and check that these two compositions each have the correct value. (If you didn't find \( f^{-1} \) in part (a), then for partial credit you can explain what the correct values for \( (f^{-1} \circ f)(x) \) and \( (f \circ f^{-1})(x) \) are.)

Problem 28. [12 points] A homeowner wants to enclose a rectangular piece of land beside a river. He must put a fence on three sides of the field (the river forms the fourth side). The field has length \( L \) and width \( W \) (see diagram below). The homeowner has enough material to build a total of 200 yards of fence. What should the width \( W \) and the length \( L \) of the field be, in order to produce the maximum possible area using 200 yards of fence?

\[
\begin{array}{c}
\text{RIVER} \\
W \\
\hline
L \\
W
\end{array}
\]
Some formula reminders

Note: these formulas are intended to help you recall some topics in the course, but they are not complete or comprehensive. You will need to know other formulas besides the ones given here, and some formulas given here may not be used in the exam.

**Interest** Let $A$ denote the balance at time $t$ of an account with initial deposit $P$ and with continuous compounding with interest rate $r$. Then $A = Pe^{rt}$.

**Equally likely outcomes**

$$P(E) = \frac{|E|}{|S|}.$$  

**Independence**

$$P(E \cap F) = P(E) \cdot P(F).$$

**Inclusion and Exclusion**

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

**Complements**

$$P(E^c) = 1 - P(E).$$

**Binomial probability**

$$P(k \text{ successes}) = C(n,k) p^k (1-p)^{n-k}, \text{ where } C(n,k) = \ldots$$

**Conditional probability**

$$P(E \cap F) = P(F) \cdot P(E|F).$$

**Expected value**

$$\mathcal{E} = w_1P(s_1) + \ldots + w_nP(s_n).$$