Math 1051
Fall 2006 Final Exam Problems

This exam contains 15 multiple-choice questions and 5 written-answer problems; no point scheme was provided.

M1. The value of \( \log_2 3 \cdot \log_5 4 \cdot \log_9 5 \) is

a) 1
b) \( \log_{(2 \cdot 3 \cdot 9)} (3 \cdot 4 \cdot 5) \)
c) \( \log_2 3 + \log_5 4 + \log_9 5 \)
d) \( \left( \log \frac{3}{2} \right) \left( \log \frac{4}{5} \right) \left( \log \frac{5}{9} \right) \)
e) 10

M2. Consider the line \( L \) defined by the equation \( y = 2x + 1 \)
The slope of a line perpendicular to \( L \) is

a) \(-2\)
b) \(-0.5\)
c) 1
d) 2
e) undefined

M3. Which of the following is true about the polynomial function \( f(x) = -x^3 + 4x^2 - 4x \)

a) The end behavior is \( x^3 \); that is, \( f(x) \simeq x^3 \) for \( |x| \) very large
b) \( f(x) \) is an odd function
c) The domain of \( f(x) \) is \( \{x | x \neq 0, x \neq 2\} \)
d) The graph of \( f(x) \) touches the \( x \)-axis at \( x = 2 \)
e) none of the above
M4. The solution of the equation \[ 3 \cdot 9^x - 3^x = 0 \] is

a) \(\{x = -1 \text{ or } x = 0\}\)
b) \(\{x = \frac{1}{3} \text{ or } x = 1\}\)
c) \(\{x = -1\}\)
d) \(\{x = 0\}\)
e) none of the above

M5. Consider the expression \[ \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} \].

An equivalent expression is

a) \(\sqrt{2}\)
b) \(\sqrt{6} + 2\)
c) \(\sqrt{3}\)
d) \(\sqrt{3} + \sqrt{2}\)
e) none of the above

M6. Consider \[ f(x) = \frac{1}{x} + 1 \]

Which from the following equals \(f \circ f\), after simplification?

a) \(\frac{2x + 1}{x + 1}\)
b) \(\frac{2x}{x + 1}\)
c) \(x - 1\)
d) 1
e) \(\frac{1}{x}\)
M7. If $A = (1, -2)$ and $B = (-1, 2)$ are two diametrically opposite points on a circle, then the center of that circle is

a) $(1, 2)$
b) $(-1, 1)$
c) $(1, -1)$
d) $(2, 1)$
e) $(0, 0)$

M8. The **range** of the function $f(x) = 5^{-x}$ is

a) all the real numbers
b) all the positive numbers
c) all the negative numbers
d) all the non-negative numbers
e) all the non-positive numbers

M9. What function is finally graphed after the following transformations are applied to the graph of $f(x) = \ln x$

(i) shift right 1 unit,
(ii) reflect about the $x$-axis, and
(iii) shift up 1 unit?

a) $\ln (x + 1) - 1$

b) $-\ln x - 1$

c) $\ln (-x + 1) - 1$

d) $\ln (x - 1) + 1$

e) $\ln \left(\frac{1}{1 + x}\right) + 1$
M10. What rational function might have the following graph?

a) \( \frac{1 - x^2}{x} \)

b) \( \frac{x^3}{1 - x} \)

c) \( \frac{1}{1 - x^2} \)

d) \( \frac{x}{1 - x^2} \)

e) \( \frac{x^2 + 2}{1 - x^2} \)

M11. \( \ln \frac{x}{e^x} \), for any \( x > 0 \), equals

a) \( \ln x \)

b) \( \ln x - x \)

c) \( e \)

d) \( \ln x + x \)

e) \( 1 \)

M12. Which of the following is true regarding the quadratic function

\[ f(x) = ax^2 + bx + c, \]

where \( a, b, \) and \( c \) are such that \( a \neq 0 \) and \( b^2 - 4ac < 0 \)?

a) The graph of \( f \) opens down and is below the \( x \)-axis.
b) The graph of \( f \) opens up and is above the \( x \)-axis.
c) The vertex of \( f \) is the \( x \)-intercept.
d) There are no \( x \)-intercepts.
e) None of the above.
M13. The number of **vertical** asymptotes corresponding to the rational function

\[ f(x) = \frac{x^3 - a^3}{x^2 - a^2} \]

is

a) zero.
b) one.
c) two.
d) three.
e) eight.

M14. The **Least Common Multiple** (LCM) of the polynomials

\[ x^2 + x - 6, \quad (x + 3)^2 (x - 2), \quad \text{and} \quad x^2 + 5x + 6 \]

is

a) \( (x + 2) + (x + 3)^2 + (x - 2) \)
b) \( (x + 2) (x + 3)^4 (x - 2)^2 \)
c) \( (x + 3)^2 (x^2 - 4) \)
d) \( (x + 2) (x + 3) (x - 2) \)
e) \( (x + 3)^2 \)

M15. When the polynomial \( x^3 + a^3 \) is divided by the polynomial \( x + a \), the **quotient** is

a) \( x^2 + a^2 \)
b) \( x^2 - ax + a^2 \)
c) \( x^2 + ax + a^2 \)
d) \( x^2 - a^2 \)
e) 0
Problem 1. A Norman window has the shape of a rectangle surmounted by semicircle of diameter equal to the width of the rectangle (see figure below). If the perimeter of the window is 10 feet, what dimensions will give maximal rectangle area?

![Diagram of a Norman window](image)

Problem 2. Let \( f(x) = \frac{2x}{3x - 1} \) and \( g(x) = \frac{2}{x} \) find:

(i) \((f \circ g)(x)\). You do not need to simplify your answer.

(ii) \( f^{-1}(x) \), the inverse of \( f(x) \). (The function \( f(x) \) is one-to-one.)

Problem 3. Find the center and the radius of the circle \( 2x^2 - 4x + 2y^2 = 0 \)

Problem 4. Solve the equation \( 7^x = 5^{3x-2} \)

Problem 5. Two cars are approaching an intersection. At time \( t = 0 \), one car is 1 mile south of the intersection and is moving at a constant speed of 3 miles/hour. At the same time, the other car is 5 miles east of the intersection and is moving at a constant speed of 2 miles/hour.

(i) Express the distance \( d \) between the two cars as a function of the time. That is, \( d = d(t) \).

(ii) Consider the function \( f(t) = d^2(t) \). What is the minimal \( f \) and when does it occur?