Math 1051
Spring 2005 Final Exam Problems

This exam contains 10 multiple-choice questions, worth 3 points each, and 7 written problems, worth 10 points each, for a total of 100 points.

M1. Which of the following lines is perpendicular to the line $y = -3x + 2$ and passes through the point $(3, 2)$?

a) $y = -3x - \frac{1}{2}$.

b) $y = 3x + 2$.

c) $y = -\frac{1}{3}x + \frac{1}{2}$.

d) $y = \frac{1}{3}x + 1$.

e) $y = x - 1$.

M2. Consider the circle given by the equation $x^2 + y^2 + y = 0$. The radius of this circle is

a) 4.

b) 2.

c) 1.

d) $\frac{1}{2}$.

e) $\frac{1}{4}$.

M3. Consider the function $f(x) = 3x^4 + 2x^2 - 1$. The graph of $f(x)$

a) is symmetric with respect to the $x$-axis.

b) is symmetric with respect to the $y$-axis.

c) is symmetric with respect to the origin.

d) has no $y$-intercept.

e) has an $x$-intercept of $\sqrt{\frac{1}{2}}$.

M4. Which of the following polynomials has a degree of 16 and touches the $x$-axis exactly 3 times?

a) $f(x) = -3x^2(x - 3)^2(x - 1)^3(x + 1)^3(x + 4)^4$.

b) $f(x) = -3(x - 3)(x - 1)^6(x + 1)^5(x + 4)^4$.

c) $f(x) = 3x^4(x - 3)^2(x - 1)^3(x + 1)^5(x + 4)^2$.

d) $f(x) = 3(x - 3)^4(x - 1)^2(x + 1)^5(x + 4)^4$.

e) None of the above.

M5. Which of the following rational functions has the oblique asymptote $y = 3x + 1$ and a $y$-intercept of 2?

a) $f(x) = \frac{3x}{x^2 - 3}$.

b) $f(x) = \frac{6x+2}{2x+1}$.

c) $f(x) = \frac{3x^2+4x+2}{x+1}$.

d) $f(x) = \frac{3x^2+2}{x}$.

e) $f(x) = \frac{3x^2+1}{x}$. 
M6. The following is a table of values of two one-to-one functions \( f(x) \) and \( g(x) \):

\[
\begin{array}{c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 \\
 \hline
 f(x) & 1 & -2 & -1 & 0 & 3 \\
 g(x) & 2 & 0 & -2 & -1 & 5 \\
\end{array}
\]

Which of the following statements is true?

a) \( f \circ g(-2) = -1 \).
b) \( g \circ f(-2) = -1 \).
c) \( f^{-1}(-2) = -2 \).
d) \( g^{-1}(-2) = -2 \).
e) None of the above statements is true.

M7. The domain of the function \( g(x) = \sqrt{\frac{x}{1-x}} \) is given by

a) \([0, \infty)\).
b) \((-\infty, 0)\).
c) \((0, 1)\).
d) \([0, 1)\).
e) all real numbers.

M8. The inverse function of \( f(x) = x + 2 \) is given by

a) \( f^{-1}(x) = \frac{1}{x} + 2 \).
b) \( f^{-1}(x) = x + \frac{1}{2} \).
c) \( f^{-1}(x) = \frac{1}{x+2} \).
d) \( f^{-1}(x) = x - 2 \).
e) \( f^{-1}(x) = -x - 2 \).

M9. The algebraic expression \( \log_2(x) + \log_4(x) \) equals

a) \( \log_4(x) \).
b) \( \log_8(x) \).
c) \( 3 \log_2(x) \).
d) \( \frac{3}{2} \log_2(x) \).
e) \( \frac{1}{2} \log_4(x) \).

M10. The value of \( e^{-2 \ln(w)} \) is

a) \( w \).
b) \( \sqrt{w} \).
c) \( w^2 \).
d) \( \frac{1}{w^2} \).
e) \( \sqrt[3]{w} \).
1. A circle $C$ is centered at $(4, 5)$ and touches the $x$-axis.
   a) Find the equation of the circle $C$.
   b) Find the $y$-intercept(s) of the circle $C$.
   c) What is the radius of a circle whose area is four times the area of the circle $C$?

2. A line $L$ passes through the points $(0, 4)$ and $(8, 0)$.

   a) Find the equation of the line $L$.
   b) A rectangle is bounded by the $x$- and $y$-axes and by the graph of the line $L$. Find the area $A$ of the rectangle as a function of $x$.
   c) What is the domain of the function $A(x)$ found in b)?
   d) For which value of $x$ is the area $A(x)$ from b) a maximum? Find the value of this maximum.

3. Consider the function $f(x) = -|x - 2| + 1$.
   a) Graph the function $f(x)$ using transformations, starting with the graph of $y = |x|$. Clearly list all the transformations that you use!
   b) Using the graph of $f(x)$ obtained in a), answer the following questions:
      i) For which value(s) of $x$ does the graph of $f(x)$ have a local maximum/minimum?
      ii) List the interval(s) on which the graph of $f(x)$ is decreasing, if any.
      iii) Find all intercepts of $f(x)$, if any.

4. Solve the following equations:
   a) $e^{-2x+3} = \frac{1}{e^{x+4}}$.
   b) $\log_2(\log_2(x + 3)) = 1$. 
5. Graph the rational function

\[ R(x) = \frac{3(x + 1)(x - 2)}{(x + 2)(x - 3)}. \]

To do so, first analyze the function \( R(x) \); in particular, find the domain of \( R(x) \), locate any \( x \)- and \( y \)-intercepts, find all vertical and horizontal or oblique asymptotes and construct a table (intervals, location of the graph with respect to the \( x \)-axis, etc.) as presented in class.

6. Consider the function

\[ f(x) = \frac{2x - 1}{x + 3}. \]

Answer the following questions about this function:

a) Find the \( y \)-intercept of the graph of \( f(x) \), if any.

b) Find the \( x \)-intercept(s) of the graph of \( f(x) \), if any.

c) Find \( f^{-1}(x) \) or explain why the function does not have an inverse.

d) Find the domain of \( f(x) \).

e) Find the range of \( f(x) \).

7. The temperature \( F \), in degrees Fahrenheit, of a dessert placed in a freezer for \( t \) hours is given by the rational function

\[ F(t) = \frac{60}{t^2 + 2t + 1}, \quad t \geq 0. \]

a) Find the temperature of the dessert after it has been in the freezer for 4 hours.

b) After how many hours in the freezer does the dessert have a temperature of 15\( ^\circ \)F?

c) What temperature will the dessert approach as \( t \to \infty \)?

d) What is the average rate of change of the temperature of the dessert during the first 4 hours in the freezer?