Math 1151 Review
September 2006

This is a collection of problems which covers the range of topics found in this course and which is representative of problems that have appeared on final exams in the past. Keep in mind that no brief list such as this one will cover everything you have studied in the course; you should be sure to review your lecture notes and homework thoroughly. Answers are provided following this list.

1. The angle $\alpha$ lies in the quadrant $\pi/2 < \alpha < \pi$ and has the value $\cos \alpha = -5/9$. Find the other five trigonometric ratios for $\alpha$.

2. Evaluate the following expressions:
   a) $\sin^{-1}(-1/3)$
   b) $\cos^{-1}(cos(5\pi/4))$
   c) $\tan^{-1}(5/8)$

3. Using what you have learned about transformations of functions, graph the function $y = -4 \sin[(\pi/3)x + (2\pi/3)]$, showing at least one period. What are the amplitude, period and phase shift of this function?

4.
   a) Find the exact value of $\cos(11\pi/12)$.
   b) Evaluate the expression $\sin 82^\circ \cos 37^\circ - \cos 82^\circ \sin 37^\circ$ without using a calculator.

5. Establish the following trigonometric identities:
   a) $(1 + \cos \theta + \sin \theta)/(1 + \cos \theta - \sin \theta) = \sec \theta + \tan \theta$
   b) $\cos(\alpha + \beta)/\cos(\alpha - \beta) = (1 - \tan \alpha \tan \beta)/(1 + \tan \alpha \tan \beta)$
   c) $2 \sin(2\theta) \cdot (1 - 2 \sin^2 \theta) = \sin(4\theta)$

6. Find the solutions within the principal circle $0 \leq \theta < 2\pi$ for the trigonometric equations:
   a) $\cos(\theta/3 - \pi/4) = \frac{1}{2}$
   b) $(\sqrt{3})\cdot\sin \theta + \cos \theta = 1$
   c) $\cos(2\theta) + \cos(4\theta) = 0$

7. A closed triangle has sides of lengths 6, 10, and 13. Find the cosine of each angle and find the area of the triangle.

8. A planned coastal highway reaches a bay which it must be diverted around, as it would be excessively expensive to span the bay with a bridge. Using the measurements shown in the diagram, what is the length of pavement required to build the three legs of the diversion? (You may treat these as straight segments.)
9. Find the complex zeroes of \( f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65 \). Write this polynomial in factored form.

10. 
   a) We have the complex numbers \( z = 1 - i \) and \( w = 1 - i/3 \). Calculate \( zw \) and \( z/w \); you may leave your answers in polar form.
   b) Find the complex fourth roots of \( 16 - 16i \); you may leave your answers in polar form.

11. Find the equation of the parabola with its focus at (-4,4) and its directrix being the line \( y = -2 \). Graph the curve.

12. Find the center, vertices, and foci of the ellipse
   \[ x^2 + 9y^2 + 6x - 18y + 9 = 0 \]. Graph the curve.

13. Find the center, vertices, foci, and asymptotes of the hyperbola
   \[ 2y^2 - x^2 + 2x + 8y + 3 = 0 \]. Graph the curve.

14. Solve, using the elimination method, the system of linear equations
   \[ \begin{align*}
   x + 4y - 3z &= -8 \\
   3x - y + 3z &= 12 \\
   x + y + 6z &= 1
   \end{align*} \]

15. Solve the system of non-linear equations
   \[ \begin{align*}
   2/x^2 - 3/y^2 + 1 &= 0 \\
   6/x^2 - 7/y^2 + 2 &= 0
   \end{align*} \]

16. Graph the solution set for the system of inequalities
   \[ \begin{align*}
   x - 2y &\leq 6, \quad 2x + y \geq 2, \quad x^2 + y^2 \leq 36
   \end{align*} \]

17. An arithmetic series of 19 terms has the sum of 361. Its final term is 46. What is the initial term of this series and the common difference between its terms?

18. Find the sum of the geometric series \( \sum_{k=1}^{\infty} 3 \cdot (-3/4)^k \).
Answers

1. \( \sin \alpha = \frac{2 - \sqrt{14}}{9} \); \( \tan \alpha = \frac{2 - \sqrt{14}}{5} \);
   \( \cot \alpha = \frac{5 - \sqrt{14}}{28} \); \( \sec \alpha = -9/5 \); \( \csc \alpha = \frac{9 - \sqrt{14}}{28} \)

2. 
   a) \(-1/3\)  
   b) \(3\pi/4\)  
   c) \((\sqrt{39})/5\)

3. amplitude: 4; period: 6; phase shift: -2; see graph below

4. 
   a) \(-[(\sqrt{2}) \cdot (1 + \sqrt{3})]/4\) or \(-[(\sqrt{2}) + \sqrt{3})]/2\), depending upon method; 
   these values are equal  
   b) \((\sqrt{2})/2\)

5. see proofs in solution set

6. 
   a) \(\theta = 7\pi/4\)  
   b) \(\theta = 0, 2\pi/3\)  
   c) \(\theta = \pi/6, \pi/2, 5\pi/6, 7\pi/6, 3\pi/2, 11\pi/6\)

7. cosines are \(233/260, 35/52, \) and -11/40; 
   the area is \(\sqrt{13311}/4 \approx 28.84\)

8. \(10 - [2 \cdot (\sqrt{3} - \sqrt{2})] \approx 9.36\) miles

9. factorization: \(2 \cdot (x - \frac{1}{2}) \cdot (x - 5) \cdot (x - [3+2i]) \cdot (x - [-3-2i])\)

10. 
    a) \(zw = (2\sqrt{2}) \cdot [\cos(17\pi/12) + i \sin(17\pi/12)]\); 
    \(\frac{z}{w} = [(\sqrt{2}/2) \cdot [\cos(\pi/12) + i \sin(\pi/12)]]\).
    b) \((16-16i)^{1/4}\): radius is \(2^{8/8}\), arguments are \(7\pi/16, 15\pi/16, 23\pi/16\) and \(31\pi/16\)

11. equation of parabola: \(12 \cdot (y - 1) = (x + 4)^2\); see graph below

12. center: \((-3, 1)\); vertices: \((-3\pm3, 1)\); foci: \((-3\pm2\sqrt{2}, 1)\); see graph below

13. center: \((1, -2)\); vertices: \((1, -2\pm\sqrt{2})\); foci: \((1, -2\pm\sqrt{6})\); 
    asymptotes: \((y + 2) = \pm[(\sqrt{2})/2] \cdot (x - 1)\); see graph below

14. \(x = 3, y = -8/3, z = 1/9\)

15. solution set: \(\{(2, \sqrt{2}), (2, -\sqrt{2}), (-2, \sqrt{2}), (-2, -\sqrt{2})\}\)

16. see graph below

17. \(a = -8, d = 3\)

18. \(s = -9/7\)
graphs

for problem 3

\[ y = -4 \sin \left( \frac{3}{4} x + \frac{2\pi}{3} \right) \]

for problem 11

\[ 12(y-1) = (x+4)^2 \]

for problem 12

\[ \text{focus} \]

\[ \text{directrix} \]

for problem 13

\[ \text{focus} \]

\[ \text{directrix} \]

for problem 16

\[ x^2 + y^2 = 36 \]

\[ 2x - y = 2 \]

\[ (2, -2) \]

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