Math 1151  
Spring 2007 Final Exam Problems

This exam contains 14 written-answer problems, solutions for the first seven (7) of which are required for credit, and the student’s choice of three (3) of the remaining seven are to be graded for credit. All problems are worth 20 points for a total of 200 points.

\[
\begin{align*}
\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\
\cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
\tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
\end{align*}
\]

Required Problems

1. (20 points) Find the exact value of the following.

\[
\cos \left( \tan^{-1} \frac{4}{3} + \cos^{-1} \frac{5}{13} \right).
\]

2. (20 points) Let \( \sin \theta = \frac{5}{13}, \quad \frac{\pi}{2} < \theta < \pi. \)

Then calculate the exact values of \( \cos \theta \) and \( \tan \theta. \)

3. (20 points) Consider the function

\( y = 4 \sin(2x - \pi). \)

(a) Find the amplitude, period, and phase shift.

(b) Graph the function and plot \( x\)-\( y\)-intercepts on the sketch.

4. (20 points) Establish the following identity.

\[
1 - \frac{\sin^2 \theta}{1 + \cos \theta} = \cos \theta.
\]

5. (20 points) Find all solutions of the following equation on the interval \( 0 \leq \theta < 2\pi. \)

\[4 \cos^2 \theta + 4 \sin \theta - 5 = 0.\]

[Hint: Use the identity \( \sin^2 \theta + \cos^2 \theta = 1.\)]
6. (20 points) Let \( x^2 - 4x + 2y^2 + 4y = 0 \)

be an equation that defines a conic section.

(a) Write the equation in the standard form.
(b) Find the coordinates of the center, foci, and vertices.
(c) Sketch the graph.

7. (20 points) Find all complex zeros of the polynomial equation

\[ x^3 + 8x^2 + 12x - 21 = 0. \]

[Hint: Use the Rational Zero Theorem and the Quadratic Formula.]

Elective Problems (choose 3 to be graded)

8. (20 points) Using the Law of Sines, solve the triangle and sketch it.

\[ \alpha = 70^\circ, \gamma = 50^\circ, b = 5 \]

9. (20 points) Find all complex zeros of the polynomial equation \( x^4 + 4x^3 + 2x^2 - x + 6 = 0. \)

10. (20 points) Write the complex number \((\sqrt{3} + i)^7\) in the standard form \((a + bi)\).

11. (20 points) Consider \( x^2 - 4x - 2y = 0. \)

(a) Find the vertex, focus, and the directrix.
(b) Sketch the graph.

12. (20 points) Solve the system of the equation

\[
\begin{align*}
3x - 6y &= 2 \\
5x + 4y &= 1
\end{align*}
\]

13. (20 points) Let \( \sin \alpha = \frac{4}{5}, \ 0 < \alpha < \frac{\pi}{2}; \ \cos \beta = \frac{1}{2}, \ 0 < \beta < \frac{\pi}{2}. \)

Find the exact values of \( \cos(\alpha + \beta) \) and \( \sin(\alpha + \beta) \).

14. (20 points) Graph the following system of inequalities.

\[
\begin{align*}
x &\geq 0 \\
y &\geq 0 \\
x + y &\geq 3 \\
2x + y &\geq 4
\end{align*}
\]