Math 2263  
Spring 2006 Final Exam Problems

This exam contains 8 multiple-choice questions, worth 15 points each, and 8 written-answer problems, worth 20 - 30 points each, for a total of 300 points.

1. Let the curve \( \vec{r}(t) \) be given by 
\[
\vec{r}(t) = \langle 2t^2 + 2, \frac{8}{3}t^{3/2}, \frac{4}{3}, 2t + 2 \rangle
\]
Then the curvature at the point corresponding to \( t = 1 \) is

(A) \( \frac{1}{18} \)

(B) \( \frac{-1}{18} \)

(C) 18

(D) \( \frac{\sqrt{2}}{18} \)

(E) none of the above.

2. The position vector of a particle is given by \( \vec{r}(t) = \langle \cos t, \sin t, t \rangle, t \in \mathbb{R} \). Then the speed of this particle is

(A) \( < -\sin t, \cos t, 1 > \)

(B) \( < -\cos t, -\sin t, 0 > \)

(C) \( \sqrt{1 + t^2} \)

(D) 1

(E) None of the above.

3. The tangent plane to the surface \( x^2 + y^2 - y + e^z + z = 2 \) at \( (1,1,0) \) has equation

(A) \( 2x - y + 2z = 1 \)

(B) \( 2x - y + z = 1 \)

(C) \( 2x + y + 2z = 0 \)

(D) \( 2x + y + 2z = 3 \)

(E) none of the above
4. A closed cylindrical can that is 10 cm high and 4 cm in diameter. If the metal in the top and bottom is 0.1 cm thick and the metal in the sides is 0.05 cm thick, the total amount of metal is approximately

(A) $2.4\pi \text{ cm}^3$

(B) $2.8\pi \text{ cm}^3$

(C) $4.8\pi \text{ cm}^3$

(D) $7.2\pi \text{ cm}^3$

(E) none of the above

5. Let $f(x, y) = x^2 + 2y^3 - 6xy - 2x + 18y + 11$. Then

(A) $f$ has a local minimum at (4, 1) and a local maximum at (7, 2).

(B) $f$ has a saddle point at (4, 1) and a local maximum at (7, 2).

(C) $f$ has a saddle point at (4, 1) and a local minimum at (7, 2).

(D) $f$ has a local maximum at (4, 1) and a local minimum at (7, 2).

(E) none of the above.

6. Changing the order of integration in the iterated integral

$$\int_{0}^{2} \int_{\frac{y}{2}}^{\frac{y^2}{2^2}+1} f(x, y) \, dy \, dx$$

leads to

(A) $\int_{0}^{5} \int_{2y}^{\frac{5}{2}-1} f(x, y) \, dx \, dy$

(B) $\int_{0}^{1} \int_{0}^{2y} f(x, y) \, dx \, dy + \int_{1}^{5} \int_{0}^{\frac{5}{2}-1} f(x, y) \, dx \, dy$

(C) $\int_{0}^{2} \int_{2y}^{\frac{5}{2}-1} f(x, y) \, dx \, dy$

(D) $\int_{0}^{1} \int_{0}^{2y} f(x, y) \, dx \, dy + \int_{1}^{5} \int_{2y}^{\frac{5}{2}-1} f(x, y) \, dx \, dy$

(E) None of the above.
7. Let $C$ denote the circle $(x - 2)^2 + (y + 3)^2 = 25$ in $\mathbb{R}^2$, oriented counterclockwise. Then the line integral

$$\int_C (-ydx + xdy) =$$

(A) $25\pi$
(B) $15\pi$
(C) $5\pi$
(D) $0$
(E) None of the above.

8. Consider the region $D$ bounded by the lines $y = 1$, $y = 3$, the hyperbola $xy = 3$, and the $y$-axis. The centroid of $D$ (i.e. the center-of-mass of $D$ for density = 1) is the point

(A) $(1, 2)$
(B) $(3, 6)$
(C) $(3\ln 3, 12\ln 3)$
(D) $(\frac{1}{\ln 3}, \frac{2}{\ln 3})$
(E) None of the above.
9. Let \( f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^6} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases} \)

(i) (15 pts) Does the limit \( \lim_{(x,y) \to (0,0)} f(x, y) \) exist?

(ii) (5 pts) Is \( f \) continuous at \((0,0)\)?

10. (20 pts) You are standing at the point \((60,100,764)\) on a hill whose shape is given by the equation
\[
z = 1000 - (0.01)x^2 - (0.02)y^2,
\]
where \(x, y, z\) are measured in meters. (The positive \(x\)-axis points east and the positive \(y\)-axis points north.)

(i) What will your rate of climb (\textit{rise over run}) be if you head southwest?

(ii) In which direction should you proceed initially in order to climb the most steeply? (Give your answer as a unit vector.)

11. (20 pts) A rectangular open-topped box is to have volume 700 in\(^3\). The material for its bottom costs 7 cents/in\(^2\), and the material for its four vertical sides costs 5 cents/in\(^2\). What dimensions will minimize the cost of the material used in constructing this box?

12. (20 pts) Find the highest and lowest points on the ellipse formed by the intersection of the cylinder \(x^2 + y^2 = 1\) and the plane \(2x + y - z = 4\).

13. (20 pts) Find the volume of the region common to the interior of the cylinders \(y^2 + z^2 = 1\) and \(x^2 + z^2 = 1\).
14. (20 pts) Evaluate
\[ \iint_R xy \, dA \]
where \( R \) is the region in the first quadrant bounded by the lines \( y = x, \ y = 3x \) and the hyperbolas \( xy = 1 \) and \( xy = 3 \).

15. (30 pts) Calculate the work done by the force field
\[ \vec{F}(x, y, z) = (x^2 + z^2, (\sin y)^2 + x^2, e^z + y^2 > \]
when a particle moves under its influence around the edge of the part of the sphere \( x^2 + y^2 + z^2 = 4 \) that lies in the first octant, in a counterclockwise direction as viewed from above.

16. (30 pts) Evaluate
\[ \iint_S \vec{F} \cdot d\vec{S} \]
where
\[ \vec{F}(x, y, z) = < z^2 x, \frac{1}{3} y^3 + \tan z, x^2 z > \]
and \( S \) is the top half of the sphere \( x^2 + y^2 + z^2 = 1 \) with outward orientation. (Hint: Try to use the Divergence Theorem).