Math 1051  
Spring 2003 Final Exam Problems

This exam contains 15 multiple-choice questions, worth 3 points each, and 6 written problems, worth 9 points each (10 for #5), for a total of 100 points.

M1. After simplification, the expression \( x + 1 - \frac{x^2 + 2x}{x + 1} \) reads

a) \( x^2 + 2x \);

b) 1;

c) \( 1 - x^2 - 2x \);

d) 2;

e) \( 1/(x + 1) \).

M2. Multiply polynomials \( (x - a)(x^3 + ax^2 + a^2x + a^3) \).

The result after simplification is

a) \( x^4 + ax^3 + a^2x^2 + a^3x \);

b) \( x^2 + 2ax + a^2 \);

c) \(-ax^3 - a^2x^2 - a^3x - a^4 \);

d) \( x^4 - a^4 \);

e) none of the above.

M3. The expression \( x^{1/2}y^{-3/2}z^{1/6} \) is equal to (assuming that \( x, y, z > 0 \))

a) \( \frac{1}{3} \cdot \frac{3}{2} \cdot \frac{1}{6} \cdot z \);

b) \( -\frac{3}{48} \cdot \frac{1}{2} \cdot \sqrt{2} \);

c) \( \sqrt{2} \cdot \sqrt{\frac{1}{2}} \cdot \sqrt{\frac{1}{6}} \);

d) \( x^{1/4}y^{1/6} - y^{5/6} \);

e) none of the above.

M4. The Least Common Multiple of the polynomials

\( (x + 1)(x - 2)(x + 3), (x + 1)^3(x - 2), (x + 3)^2(x + 1) \)

is

a) \( (x + 1)^3(x - 2)^2(x + 3)^2 \);

b) \( (x + 1)^4(x - 2)^3(x + 3)^3 \);

c) \( (x + 1)(x - 2)(x + 3) \);

d) \( (x + 1)^4 + (x - 2)^3 + (x + 3)^4 \);

e) \( (x + 1), (x - 2), (x + 3) \).
M5. Divide $x^4 - x + 1$ by $x^2 + 1$. The quotient after division is
   a) $x^2 + x$;
   b) $x^3 + x^2 + x + 1$;
   c) $x^3 - x$;
   d) $x^3 - 3$;
   e) $x^3 - x^2 - x - 1$.

M6. Find the midpoint $M$ of the segment $P_1P_2$, $P_1 = (3, 1)$, $P_2 = (-2, 4)$. The answer is
   a) $M = (5/2, -3/2)$;
   b) $M = (1/2, 5/2)$;
   c) $M = (5/2, 1/2)$;
   d) $M = (1/3, 2/3)$;
   e) $M = (1, 1)$.

M7. If a leg of a right triangle has length 3 and the hypotenuse has length 4, then the other leg will have length
   a) $\sqrt{5}$;
   b) 5;
   c) $\sqrt{7}$;
   d) 7;
   e) none of the above.

M8. Consider functions $f(x) = \frac{1}{x^2 + 4x + 1}$, $g(x) = \frac{1}{x} + 3$. Which from the following is equal to $(g \circ f)(x)$ after the simplification?
   a) $\frac{1}{x^2 + 4x + 1} + 3$;
   b) $\left(\frac{1}{x} + 3\right)\left(x^2 + 4x + 1\right)$;
   c) $(x + 2)^2$;
   d) $x^2 + 4x + 1$;
   e) $\frac{x^2 + 4x + 1}{x} + 3$.

M9. If $(1, b)$ is a point on the graph of $y = x^2 - 1$, what is $b$?
   a) 0;
   b) $-1$;
   c) 1;
   d) $b^2 - 1$;
   e) 2.
M10. The Average Rate of Change of the function \( f(x) = 1 + x^2 \) from \( x = 1 \) to \( x = 3 \) is

a) 1;
b) \( f(0) \);
c) \( f(x) - f(0) \);
d) 5;
e) 4.

M11. Which from the following is true about the rational function \( R(x) = \frac{3x^4 - 2x^3 + x + 1}{x^3 - 1} \)?

a) \( R(x) \) has a horizontal asymptote \( y = 0 \);
b) \( R(x) \) has an oblique asymptote \( y = 3x - 2 \);
c) \( R(x) \) has an oblique asymptote \( y = 1/x \);
d) \( R(x) \) has a horizontal asymptote \( y = 3 \);
e) none of the above.

M12. The value of \( \frac{\log_b(y^4)}{\log_b(x^2)} \) is equal to (assuming \( a, b > 0, a, b \neq 1, \) and \( y > 0 \)).

a) \( 2 \log_b y \);
b) \( \log_b(y^4/x^2) \);
c) \( \log_b(y) \log_b(x^2) \);
d) \( \log_b(y^4/x^2) \);
e) none of the above.

M13. The inverse function \( f^{-1}(x) \) to the function \( f(x) = 1/x \) is

a) \( x^2 \);
b) \( 1/\sqrt{x}, x > 0 \);
c) \( \sqrt{x}, x > 0 \);
d) \( 1/x, x \neq 0 \);
e) none of the above.

M14. Which from the following statements is true?

a) \( e^\pi = N \) is equivalent to \( \ln N = \pi \);
b) \( \pi^\pi = M \) is equivalent to \( \ln M = \pi \);
c) \( M^N = e \) is equivalent to \( \ln N = \ln M \);
d) \( Z^e = e \) is equivalent to \( \ln Z = y \);
e) \( W^y = y \) is equivalent to \( \ln W = y \).

M15. The domain of the function \( f(x) = \log_{1-x}(x + 1) \) is

a) \( x \neq -1 \) and \( x \neq 1 \);
b) \( x > 1 \) or \( x < -1 \);
c) \( 1 < x < -1 \) and \( x \neq 0 \);
d) \( x \neq -1, x \neq 0, \) and \( x \neq 1 \);
e) \( x > 0 \).
1. Simplify the expression

\[ 1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{x}}} \]

2. Determine the domain of the function

\[ f(x) = \frac{1}{\sqrt{x} - 2} + \ln(x - 3). \]

3. Find the center and the radius of the circle

\[ x^2 + y^2 - 6x + 8y = 0. \]

4. What function is finally graphed after the following transformations are applied to the graph of

\[ f(x) = \sqrt{x} + x \]

a) reflect about \( Y \)-axis;
b) compress horizontally by a factor of 2;c) shift left 1 unit.

5. Graph the rational function

\[ R(x) = \frac{x^3 - 27}{x^2 - 2x - 3} \]

6. Solve logarithmic equation

\[ \log_4(x + 3) + \log_4 x = 1. \]