Math 1142
Spring 2003 Final Examination

This examination contains 9 multiple-choice problems, worth 10 points, each, and 6 written-answer problems, worth 30 to 40 points each, for a total of 300 points.

1. Which one of the following statements is true?
   A) \( \lim_{x \to 0} \frac{|x|}{x} = 0 \)
   B) \( \lim_{x \to 0} \frac{|x|}{x} = 1 \)
   C) \( \lim_{x \to 0} \frac{|x|}{x} = -1 \)
   D) \( \lim_{x \to 0} \frac{|x|}{x} \) does not exist
   E) None of the statements above is true.

2. The equation of the tangent line to the graph of the function \( y = \frac{1}{x} + x \) at the point \((1, 2)\) is:
   A) \( y = 2 \)
   B) \( y = 2x \)
   C) \( y = \frac{1}{2}x + \frac{3}{2} \)
   D) \( y = (\ln(2) + 2)x - \ln(2) \)
   E) \( y - 2 = \left( -\frac{1}{x^2} + 1 \right) (x - 1) \)

3. The maximum and minimum values taken by the function \( f(x) = 2x^3 - 3x^2 - 12x + 5 \) on the interval \([-3, 3]\) are:
   A) 2 and -1
   B) 12 and -15
   C) 12 and -40
   D) -1 and -3
   E) 17 and -15
4. Suppose that \( f(x) = e^x \ln(x) + x^\frac{1}{3} - 2 - \frac{1}{x^2 + 1} \). Then, the derivative of \( f(x) \) is:

A) \( xe^x + e^x \ln(x) + \frac{1}{3} x^\frac{2}{3} + \frac{2x}{x^2 + 1} \)

B) \( \frac{1}{3} x^\frac{2}{3} + \frac{e^x}{x} + x \ln(x) + \frac{x}{(x^2 + 1)^2} \)

C) \( \frac{e^x}{x} + e^x \ln(x) + 3x^\frac{2}{3} + \frac{x}{2(x^2 + 1)^2} \)

D) \( \frac{e^x}{x} + e^x \ln(x) + \frac{1}{3} x^{-\frac{2}{3}} + \frac{2x}{(x^2 + 1)^2} \)

E) \( e^{x^{-1}} + e^x \ln(x) + \frac{1}{3} x^{-\frac{2}{3}} + \frac{2x}{(x^2 + 1)^2} \)

5. Let \( f(x) = \frac{x}{x - 1} \) be a function defined so long as \( x \neq 1 \). Then, \( f''(x) \) is:

A) \( x^2(x - 1)^{-2} \)

B) \( x(x - 1)^{-2} \)

C) \( 2(x - 1)^{-3} \)

D) \( x(x - 1) \)

E) \( f''(x) \) does not exist

6. The value of \( \int_1^3 \frac{\ln(x)}{x} \, dx \) is:

A) \( \ln(3) \)

B) \( (\ln(3))^2 \)

C) \( 2 - \ln(3) \)

D) \( \frac{e^4}{3} + 1 \)

E) \( - (\ln(3))^2 \)
7. The average value of \( f(x) = (x - 1)^3 \) on the interval \([0, 2]\) is:

A) \(-\frac{3}{4}\)
B) 0
C) \(\frac{3}{8}\)
D) \(\frac{1}{2}\)
E) \(\frac{2}{3}\)

8. Which of the following statements is true?

A) \(\int_0^\infty 5e^{-5x} \, dx\) does not converge.
B) \(\int_0^\infty 5e^{-5x} \, dx = 1\)
C) \(\int_0^\infty 5e^{-5x} \, dx = 5\)
D) \(\int_0^\infty 5e^{-5x} \, dx = e^{-5}\)
E) \(\int_0^\infty 5e^{-5x} \, dx = -5\)

9. If \( f(x, y) = (x^2 + y^2)e^{xy} \), then \( \frac{\partial^2 f}{\partial x \partial y} = \)

A) \((3x^2 + 3y^2 + x^3y + xy^3)e^{xy}\)
B) \((2 + 4xy + x^2y^2 + y^4)e^{xy}\)
C) \((2 + 4xy + x^2y^2 + x^4)e^{xy}\)
D) \(2x^2e^{xy}\)
E) \(2ye^{xy}\)
10. Consider the function \( y = x^4 - 4x^3 + 12 \).

   A) Find the critical points . (6 points)
   B) Which of the critical points are, respectively, local maxima, local minima, or inflection points? (8 points)
   C) Where is the function increasing? Where does it decrease? (8 points)
   D) Where is the graph concave up? Where is it concave down? (8 points)
   E) Sketch the curve for \(-1 \leq x \leq 4\). (10 points)

11. The equation \( x^2 - 3xy + y^2 = -1 \) defines a curve in the plane.

   A) At a point where this curve is the graph of a function \( y \) of \( x \), use implicit differentiation to find \( \frac{dy}{dx} \). (20 points)
   B) Use the formula that you obtained in part A) to find the equation of the tangent line to this curve at the point \((2, 1)\). (15 points)

12. Find the area of the region bounded above by \( y = xe^{-x^2} \), \( x \geq 0 \), and below by the line \( y = 0 \). (30 points)

13. A particle travels along the X-axis. Its position at time \( t \), \( 0 \leq t \leq 10 \), is given by

   \[ x(t) = 360,000 - \frac{4}{10}(360,000 - 36,000r + 1800r^2 - 60r^3 + r^4) \].

   Find the velocity of the particle at time \( t \), \( 0 \leq t \leq 10 \). Then show that the particle starts from the origin with velocity zero and travels to the right for all \( t \), \( 0 < t \leq 10 \). (35 points)

14. The function \( f(x) = \frac{x^4}{625} \) is a probability density function on the interval \([0, 5]\).

   A) Find the mean of this probability density function. (15 points)
   B) Find the standard deviation of this probability density function. (20 points)

15. The weights of the students in a certain college are normally distributed with mean 155 lbs. and standard deviation 35 lbs. What percentage of the student body is expected to have weights between 145 lbs. and 160 lbs.? You may use the shortened table of areas under the standard normal distribution curve given below.

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(35 points)