Math 1142
Spring 2005 Final Examination

This examination contains 12 multiple-choice questions, worth 12 points each, and 6 written-answer problems, worth 25 to 31 points each, for a total of 300 points.

1. \( \lim_{x \to \infty} \frac{x+1}{2x^2+4} = \)
   
   A. 0
   
   B. 1
   
   C. \(\frac{1}{2}\)
   
   D. \(\frac{1}{4}\)
   
   E. \(\infty\)

2. \( f(x) = \frac{2x+3}{x^2+1} \). What is \( f'(1) \)?
   
   A. \(\frac{5}{2}\)
   
   B. 2
   
   C. 1
   
   D. \(-\frac{3}{2}\)
   
   E. \(-\frac{5}{2}\)

3. If \( y = \sqrt{u} \), and \( u = (2x^2 + 5)^3 \), then \( \frac{dy}{dx} = \)
   
   A. \(\frac{3}{2}(2x^2 + 5)\)
   
   B. \(\frac{3}{2} \sqrt{2x^2 + 5}\)
   
   C. \(6x \sqrt{2x^2 + 5}\)
   
   D. \(\sqrt{3}(2x^2 + 5)\)
   
   E. \((2x^2 + 5)^{\frac{3}{2}}\)
4. What is the slope of the tangent line to the curve \( y = 2\left(\frac{1}{x} + 1\right)(x^2 - 5x + 3) \) at the point where \( x = 1 \)?

A. 6
B. 2
C. -2
D. -4
E. -10

5. \( f(x) = \begin{cases} 
\frac{(x^2 - 9)}{(x - 3)} & \text{if } x < 3 \\
\frac{x - 3}{(x - 3)} & \text{if } x \geq 3
\end{cases} \)

Find \( \lim_{x \to 3^-} f(x) \).

A. 6
B. 3
C. 0
D. -3
E. Does not exist

6. Evaluate the definite integral \( \int_1^2 x(x^2 - 1)^4 \, dx \).

A. \( \frac{1}{10} \)
B. \( \frac{81}{10} \)
C. 81
D. 162
E. \( \frac{243}{10} \)
7. The average value of \( f(x) = x(x - 2) \) over \( 0 \leq x \leq 2 \) is
   
   A. \(-4\)
   
   B. \(-\frac{4}{3}\)
   
   C. \(-\frac{2}{3}\)
   
   D. 0
   
   E. 1

8. Let \( f(x) = e^{x^3 - 3x} \). Which of the following is true?
   
   A. \( f(x) \) is increasing everywhere.
   
   B. \( f(x) \) is decreasing everywhere.
   
   C. \( f(x) \) is increasing on \((-\infty, -1)\) and \((1, \infty)\), and decreasing on \((-1, 1)\).
   
   D. \( f(x) \) is increasing on \((-1, 1)\), and decreasing on \((-\infty, -1)\) and \((1, \infty)\).
   
   E. \( f(x) \) is increasing on \((0, 3)\), and decreasing on \((-\infty, 0)\) and \((3, \infty)\).

9. Let \( f(x) = x^4 - 4x^3 + 10 \). Which of the following is true?
   
   A. \( f(x) \) is concave upward on \((3, \infty)\), and concave downward on \((-\infty, 3)\).
   
   B. \( f(x) \) is concave upward on \((-\infty, 0)\) and \((3, \infty)\), and concave downward on \((0, 3)\).
   
   C. \( f(x) \) is concave upward on \((0, 2)\), concave downward on \((-\infty, 0)\) and \((2, \infty)\).
   
   D. \( f(x) \) is concave upward on \((-\infty, 0)\) and \((2, \infty)\), and concave downward on \((0, 2)\).
   
   E. \( f(x) \) is concave upward on \((0, 3)\), and concave downward on \((-\infty, 0)\) and \((3, \infty)\).
10. If \( f(x, y) = \ln(x^2 + y^2 + 1) \), the partial derivative \( f_x \) evaluated at \( x = 1, \ y = 0 \) is

A. 0

B. 1

C. 2

D. \( \ln 2 \)

E. \( \ln 3 \)

11. Consider the function \( f(x, y) = x^2 + 2y^2 - xy + 6 \). Which of the following is true?

A. \( f(x, y) \) has a saddle point at \((1, 1)\).

B. \( f(x, y) \) has a relative maximum at \((0, 6)\).

C. \( f(x, y) \) has a saddle point at \((0, 6)\).

D. \( f(x, y) \) has a relative maximum at \((0, 0)\).

E. \( f(x, y) \) has a relative minimum at \((0, 0)\).

12. The equation \( x^2 - xy + 2y^2 = 2 \) defines \( y \) implicitly as a function of \( x \).

Find \( \frac{dy}{dx} \) by implicit differentiation.

A. \( \frac{dy}{dx} = \frac{y-2x}{4y-x} \)

B. \( \frac{dy}{dx} = \frac{y+2x}{2y-x} \)

C. \( \frac{dy}{dx} = 2x - y \)

D. \( \frac{dy}{dx} = \frac{2x}{1+4y} \)

E. \( \frac{dy}{dx} = \frac{2x}{1-4y} \)
13. (25 pts) Find the area of the region enclosed by the curves \( y = x^2 - 2x \) and \( y = -x^2 \).

14. (25 pts) Use Simpson's rule with \( n = 4 \) to approximate the definite integral
\[
\int_{-2}^{2} \frac{4}{x^2 + 4} \, dx.
\]
(Simpson's Rule:
\[
\int_{a}^{b} f(x) \, dx \approx \frac{\Delta x}{3} \left[ f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \cdots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1}) \right],
\]
where \( \Delta x = \frac{b-a}{n} \) and \( x_i = a + (i-1)\Delta x \).

15. (25 pts) Find the particular solution of the differential equation satisfying
\[
\frac{dy}{dx} = xe^{-2y}, \quad y = 0 \text{ when } x = 0.
\]

16. (25 pts) Find the absolute maximum and absolute minimum value of
\( f(x) = 2x + \frac{8}{x} \) for \( 1 \leq x \leq 3 \).

17. (25 pts) Find two numbers \( x \) and \( y \) so that \( x + 2y = 36 \) and \( xy \) is as large as possible.

18.
(a) (15 pts) Use the formula
\[
\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| u + \sqrt{a^2 + u^2} \right| + C
\]
to find
\[
\int \sqrt{100 + 25x^2} \, dx.
\]
(b) (16 pts) Use integration by parts to find the integral
\[
\int x \ln x^3 \, dx.
\]