Math 1151  
Fall 2005 Final Examination  
This examination contains 15 written-answer questions.  
worth 20 points each, for a total of 300 points.

1. (20 points) A parabola has focus at \((0, 0)\). If the directrix of the parabola is the line \(x = -1\), find the equation and graph it.

2. (20 points) Find the center, foci, and the vertices of the ellipse whose equation is \[4x^2 - 16x + y^2 - 6y = -9.\]

3. (20 points) Find the equation of the hyperbola that has one of the foci at \((0, 5)\) and the two vertices at \((0, 4)\) and \((0, -1)\), respectively. Graph this equation and the two asymptotes.

4. (20 points) Use the Rational Zeros Theorem to find all the real zeros of the following polynomial function \[f(x) = x^3 + x^2 - 8x + 4.\]
   Use the zeros to factor \(f(x)\) over the real numbers.

5. (a) (10 points) If \(z = i\), find \(z^{101}\).
   (b) (10 points) Find the complex fourth roots of \(z = 1 + i\).

6. (20 points) Let \(f(x)\) be the following polynomial with real coefficients \[f(x) = x^3 - 7x^2 + 17x - 15.\]
   If \(2 + i\) is zero of \(f\), find the remaining zeros and write \(f\) in factored form.

7. (20 points) Find the area of the sector of a circle of radius 2 meters formed by an angle of 10 degrees.

8. (20 points) The length of an arc of a circle is 240 meters. If this arc is determined by a central angle of \(30^\circ\), find the radius of the circle.

9. (20 points) Find the area of the triangle having \(a = 2\), \(b = 3\), and \(c = 4\).
10. (20 points) Solve the triangle: \(a = 4, b = 6, \text{ and } \gamma = 60^\circ.\) 
(Recall that \(\gamma\) stands for the angle opposite to the side \(c\).)

11. (20 points) Consider the equation 
\[
\sin^4(x) + \cos(2x) = 1.
\]
Give a general formula for all solutions. List four of them.

12. (20 points) Find the amplitude, period, and the phase shift of the function 
\[y = -2\sin(2\pi x - 4).\]
Sketch the graph of this function. Label at least 5 key points on the graph of \(y\).

13. (20 points) Find the exact value of 
\[
\cos(\tan^{-1}(1/3)) + \sin^{-1}(1) .
\]

14. (20 points) Graph the system of inequalities 
\[
y \geq x^2 - 9 \\
3y - x - 3 \leq 0
\]
Label all points of intersection.

15. (20 points) Find the sum 
\[
\sum_{k=1}^{2005}(2 \cdot k^2 + 3 \cdot k).
\]