Math 2263  
Fall 2005 Final Exam Problems

This exam contains 12 multiple-choice questions, worth 15 points each, and 8 written-answer questions, worth 25 - 30 points each, for a total of 400 points.

[Note: #20 was missing from the original scan of this exam.]

1. Let \( \mathbf{r}(t) = t\mathbf{i} + (\cos 2t)\mathbf{j} + (\sin 2t)\mathbf{k} \). Then the curvature at the point corresponding to \( t = 0 \) is

   (A) 0  
   (B) 1  
   (C) 4  
   (D) 4/5  
   (E) \( \frac{4}{5} \)

2. Let \( f(x, y) = xe^{xy} \). Then \( \frac{\partial^2 f}{\partial x \partial y}(0, 0) \) is

   (A) 0  
   (B) 1  
   (C) -1  
   (D) e  
   (E) None of the above.

3. An equation for the tangent plane to the surface \( xz^2 - 2y^3 = -1 \) at the point \((1, 1, 1)\) is

   (A) \( x^2(x - 1) - 6y^2(y - 1) + 2xz(x - 1) = 0 \)  
   (B) \( x - 1 = (x - 1) - 6(y - 1) \)  
   (C) \( x - 6y + 2z = -1 \)  
   (D) \( x - 6y + 2z = -3 \)  
   (E) \( (x - 1) + 6(y - 1) + 2(z - 1) = 0. \)

4. The function \( f(x, y) = x^2 + y^3 + 6(x + y^2) - 15y \) has, at the point \((-3, -3)\),

   (A) A local minimum  
   (B) A local maximum  
   (C) A saddle point  
   (D) A critical point where the second derivative test is inconclusive  
   (E) No critical point of any type.
5. \( \int_0^a \int_0^{\pi/2} f(x, y) dy dx \) is, for every continuous function \( f(x, y) \),

(A) \( \int_0^a \int_0^{\pi/2} f(x, y) dx dy \)

(B) \( \int_0^a \int_0^{\pi/2} f(x, y) dy dx \)

(C) \( \int_0^a \int_{\pi/2}^{\pi} f(x, y) dx dy \)

(D) \( \int_0^a \int_{\pi/2}^{\pi} f(x, y) dy dx \)

(E) \( \int_0^a \int_0^{\pi/2} f(x, y) dx dy \)

6. The domain of integration for the integral

\[ \int_{\sqrt{2}}^{\sqrt{2}} \int_{\sqrt{2}-z^2}^{\sqrt{2}-z^2} \int_{\sqrt{2-z^2}}^{\sqrt{2-z^2}} f(x, y, z) dz dy dx \]

is

(A) The solid in \( \mathbb{R}^3 \) bounded below by a paraboloid and above by a sphere

(B) The solid in \( \mathbb{R}^3 \) bounded below by a cone and above by a sphere.

(C) The solid in \( \mathbb{R}^3 \) bounded below by a paraboloid and above by a cone.

(D) The solid in \( \mathbb{R}^3 \) bounded below by the \( (x, y) \)-plane, on the sides by a cylinder and above by a cone.

(E) The solid in \( \mathbb{R}^3 \) bounded below by a cone and above by a paraboloid.

7. The volume of the solid in \( \mathbb{R}^3 \) consisting of points \( (x, y, z) \) lying inside the cylinder \( x^2 + y^2 = 9 \) and outside the cone \( (z - 3)^2 = x^2 + y^2 \) is

(A) \( 36\pi \)

(B) \( 54\pi \)

(C) \( 6\pi \)

(D) \( 9\pi \)

(E) \( 18\pi \)

8. The vector field \( \vec{F}(x, y) = (6\alpha xy + 4\beta y^2 + 10x^2)\vec{i} + (12x^2 + 4xy - 9y^2)\vec{j} \) is conservative when

(A) \( \alpha = 1, \beta = 0 \)

(B) \( \alpha = 5, \beta = 3 \)

(C) \( \alpha = -4, \beta = 1/2 \)

(D) \( \alpha = -4, \beta = -1/2 \)

(E) \( \alpha = 4, \beta = 1/2 \)
9. Evaluate $\int \int_C x \, dx - x^2 y^2 \, dy$ where $C$ is the triangle with vertices $(0, 0), (1, 1), (0, 1)$ and $C$ is oriented in the counterclockwise direction.

(A) $11/5$
(B) $1/5$
(C) $-1/3$
(D) $-1/5$
(E) $-1$

10. The surface area of the portion of the sphere $x^2 + y^2 + z^2 = 25$ that lies above the triangle with vertices $(0, 0), (3, 0), (0, 2)$ is

(A) $\int_0^3 \int_0^{(-2/3)z+2} \frac{2z^2+y^2-25}{\sqrt{z^2+y^2-25}} \, dy \, dz$
(B) $\int_0^3 \int_0^{(-2/3)z+2} (25 - x^2 - y^2)^{3/2} \, dy \, dx$
(C) $\int_0^3 \int_0^{(5/\sqrt{25-z^2-y^2})} \, dy \, dx$
(D) $\int_0^3 \int_0^{(-2/3)z+2} \frac{5}{\sqrt{25-z^2-y^2}} \, dy \, dx$
(E) $\int_0^2 \int_0^{(-3/2)y+3} \frac{25}{\sqrt{25-z^2-y^2}} \, dx \, dy$

11. Let $\vec{F}(x, y, z) = x^2 \vec{i} + (-yz) \vec{j} + xz \vec{k}$. Then curl $\vec{F}$ is

(A) $2xy \vec{i} - xz \vec{j} + x \vec{k}$
(B) $-y \vec{i} + xz \vec{j} + x^2 \vec{k}$
(C) $-y \vec{i} + x \vec{j} - x^2 \vec{k}$
(D) $y \vec{i} - x \vec{j} - x^2 \vec{k}$
(E) $y \vec{i} + x \vec{j} + x^2 \vec{k}$

12. Let $E$ be the solid cone bounded below by $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 2$. Let $\vec{F}(x, y, z) = z \vec{i} + y \vec{j} + z \vec{k}$. Then $\iiint_E \text{div} \vec{F} \, dV$ is

(A) $\frac{3}{2} \pi$
(B) $8 \pi$
(C) $16 \pi$
(D) $\frac{16}{3} \pi$
(E) $0$
13. (25 pts) Find the length of the parametrized curve \( \mathbf{r}(t) = 15t\mathbf{i} + \frac{400t^3}{3}\mathbf{j} + \frac{5t^2}{2}\mathbf{k} \) from \((0,0,0)\) to \((15, \frac{400}{3}, 5/2)\).

14. (25 pts) If \( z = x^2e^{\sqrt{x^2+y^2}} \) and \( x = r\cos \theta, \ y = r\sin \theta \), find \( \frac{\partial z}{\partial r} \) and \( \frac{\partial z}{\partial \theta} \).

15. (30 pts) Using Lagrange multipliers find the point on the line where the planes \( x + y + z = 6 \) and \( 2x - 3y + z = 0 \) meet which is closest to the origin.

16. (25 pts) Evaluate the double integral \( \iint_D \frac{1}{x^2 + y^2} \, dA \) where \( D \) is the region \( \{(x, y), 4 \leq x^2 + y^2 \leq 9\} \).

17. (30 pts) Evaluate the integral \( \iiint_D e^\sqrt{x^2+y^2+z^2} \, dV \) where \( D \) is the solid in the first octant determined by \( 4 \leq x^2 + y^2 + z^2 \leq 9 \).

18. (30 pts) Using the change of variables \( x = \frac{u+v}{2}, \ y = u-v \) evaluate the double integral
\[
\iint_D e^{\frac{u+v}{2}} \, dA
\]
where \( D \) is the trapezoid in the first quadrant with vertices \((2, 0), (4, 0), (0, 4), (0, 8)\).

19. (25 pts) Evaluate the surface integral \( \iint_S xz^2 \, dS \) where \( S \) is the portion of the cone \( z = \sqrt{x^2+y^2} \) between the planes \( z = 2 \) and \( z = 4 \).

[ #20 is missing from the original scan of this final exam.]