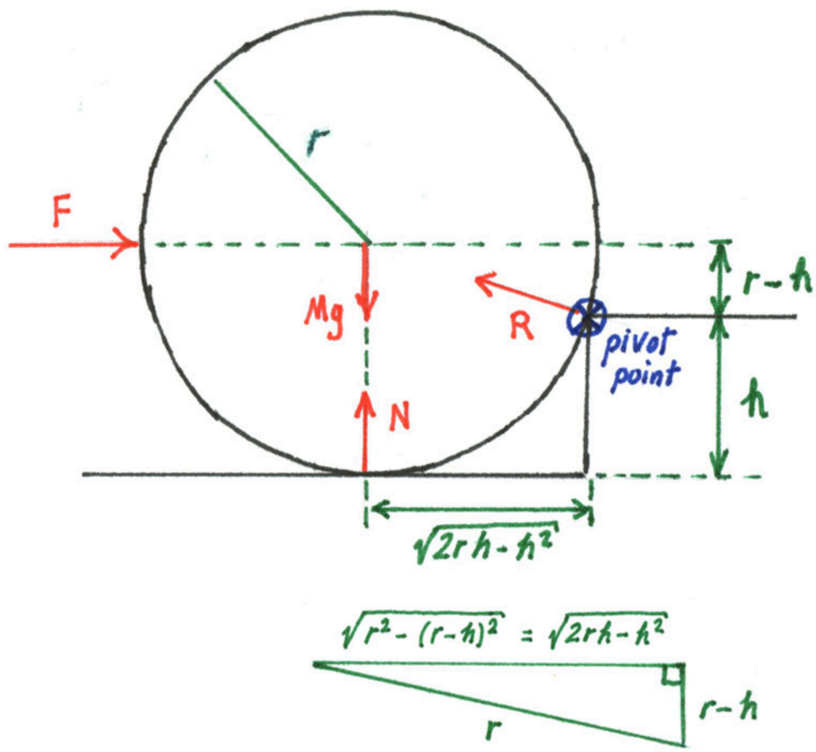
Solutions for Physics 1301 Course Review (Problems 10 through 18)

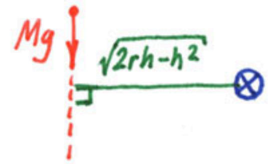
10)

a) When the bicycle wheel comes into contact with the step, there are four forces acting on it at that moment: its own weight, Mg; the normal force upward from the ground, N; the applied force, F; and the reaction force from the point of contact with the step, R. If a large enough force is applied (and in the right place, as we shall see), the wheel will pivot about the edge of the step and lift up over it. For the minimal amount of force that would just make this happen, the wheel would be very nearly in static equilibrium. It would just begin to lift off the ground, causing the normal force N to drop to zero. The reaction force R acts at the contact point where the wheel is pivoting, so the moment arm along which it acts is zero, hence the torque it produces on the wheel is also zero.



So only two of the aforementioned forces produce a torque on the wheel and only one of these is unknown, so the torque equation alone will suffice to determine the value of F. The magnitude of a torque $\vec{\tau}$ is given by $|\vec{\tau}| = |\vec{r} \times \vec{F}| = rF\sin\theta$, which is equivalent to the magnitude of the force F times the perpendicular (or closest) distance, $r\sin\theta$, from the pivot point to the "line of action" of the force. Thus we find:

torque due to weight – $\tau_W = +Mg \cdot \sqrt{2rh - h^2}$



torque due to applied force – $\tau_F = -F \cdot (r - h)$

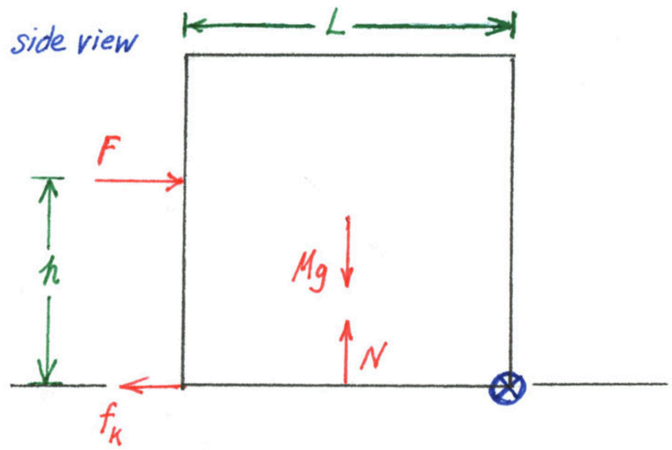
With the wheel *just about* to pivot around the step, the net torque is exactly zero, so the minimum applied force needed to lift the wheel over the step is given by

$$+Mg \cdot \sqrt{2rh - h^2} \; - \; F \cdot (r - h) \; = \; 0 \quad \Rightarrow \quad F \; = \; Mg \left(\frac{\sqrt{2rh - h^2}}{r - h} \right) \quad .$$

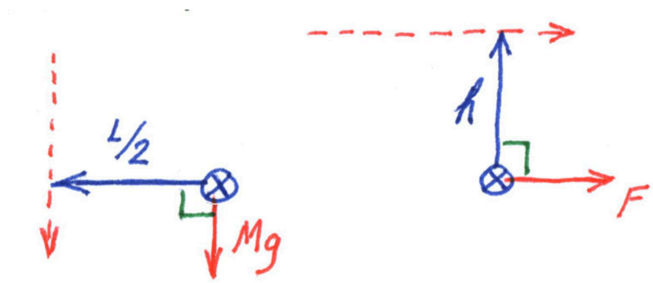
The form of this result appear reasonable. When there is no step ($h = 0$) , the horizontal force required to lift the wheel becomes $F = 0$. Somewhat less trivially, we also find that it is impossible to lift the wheel, with a horizontally applied force, over a step with a height equal to or greater than the wheel's radius ($F \rightarrow \infty$ as $r \rightarrow h$) .

Notice that the moment of inertia, I, of the wheel does not enter into the calculation for F. This indicates that the applied force required would be the same if the wheel were a uniform solid disk of the same mass and radius.

b) The force F is applied at a height h above the floor along a vertical midline of one face of the cube (this is done so that the cube pivots up off the floor without turning to one side or the other).



It is possible to push the cube hard enough in this way so that it will pivot forward on its leading edge, rather than simply remaining at rest (under static friction) or sliding along the floor (with kinetic friction). As the cube is about to pivot, the normal force from the floor drops to zero. There will then be two torques acting on the cube, since the direction of the friction force is parallel to its moment arm, making the torque due to friction zero:



torque due to weight of cube –

$$\tau_W \;=\; +Mg \cdot (\tfrac{1}{2}L) \cdot \sin 90^\circ$$

(the dashed arrows represent the "lines of actions" of each force)

torque due to applied force –

$$\tau_F \;=\; -F \cdot h \cdot \sin 90^\circ$$

So the cube is on the verge of tipping over when the net torque from these force is *just* equal to zero, or

$$\frac{1}{2}MgL \;-\; Fh \;=\; 0 \quad \Rightarrow \quad h \;=\; \left(\frac{Mg}{2F}\right)L \quad .$$

If the force is applied at a point higher above the floor than this, the cube will pitch forward and tip over, rather than slide along the floor (or possibly remain at rest if the coefficient of static friction, μ_s , is large enough).

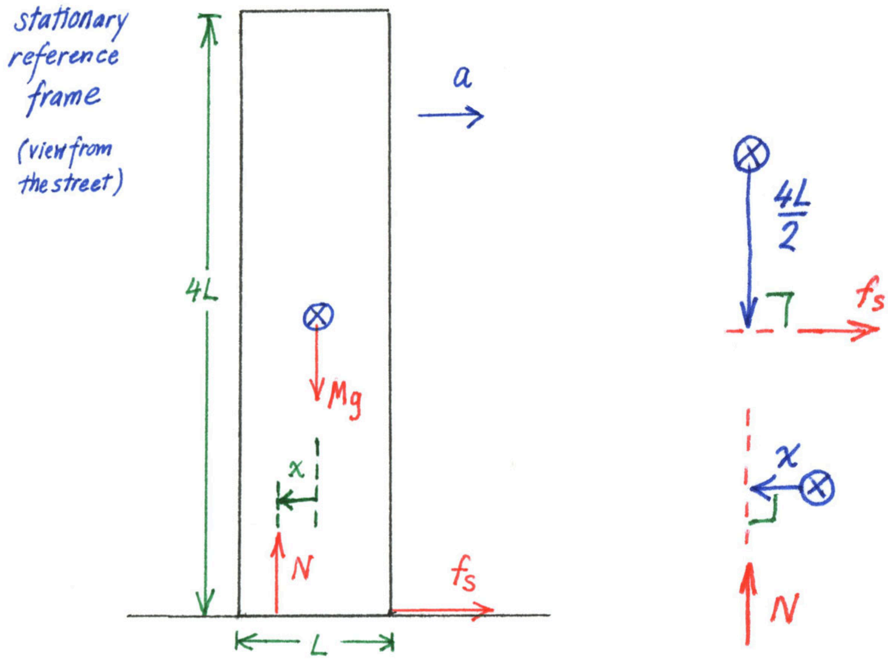
Note that as F grows very large, h becomes quite small: a very strong applied force delivered very low on the cube could still topple it. On the other hand, since the force cannot be applied any higher above the floor than $h = L$, we find that

$$h \;=\; \left(\frac{Mg}{2F}\right)L \;\le\; L \quad \Rightarrow \quad \frac{Mg}{2F} \;\le\; 1 \quad \Rightarrow \quad F \;\ge\; \frac{Mg}{2} \quad ;$$

this tells us that a force less than $\frac{1}{2}Mg$ applied anywhere *cannot* cause the cube to tip over.

c) We can examine this situation from either of two reference frames, one which is stationary, from which to watch the truck accelerating, and the other *on* the accelerating truck. Both analyses should give the same result.

From the stationary viewpoint, it is the static friction between the bottom of the stack of cubes and the truck bed that causes the stack to accelerate along with the truck. So long as the static friction can hold, $f_s \;=\; Ma$, M being the total mass of the cubes. The stack does not accelerate in the vertical direction, so $N - Mg \;=\; 0 \;\;\Rightarrow$ $N = Mg$. We now look at the net torque about the center of mass of the stack. The weight force effectively acts from this point, so the moment arm for this torque is zero, making the torque due to the weight equal to zero.



The other two torques are

torque due to friction –

$$\tau_f \;=\; +\tfrac{1}{2}(4L)\cdot f_s \cdot \sin 90^\circ \;=\; +2 f_s L \;=\; +2MaL$$

torque due to normal force –

$$\tau_N \;=\; -x \cdot N \cdot \sin 90^\circ \;=\; -Nx \;=\; -Mgx$$

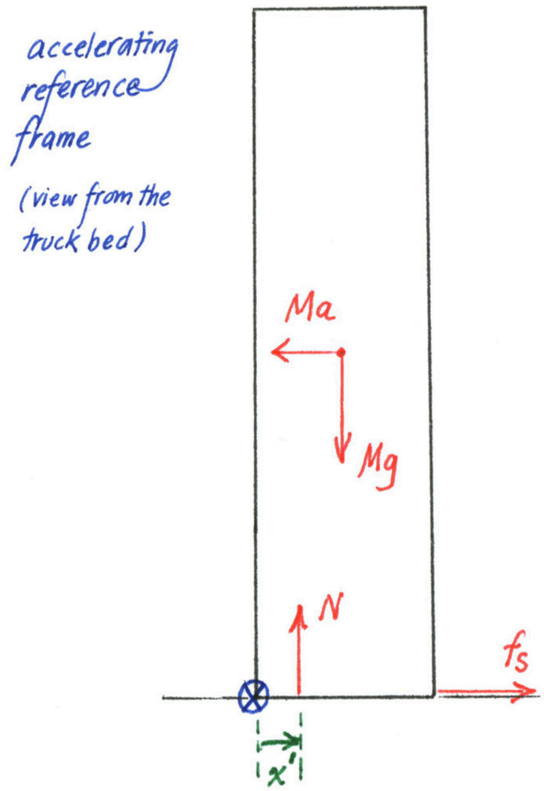
(note that for an accelerating object, the effective point at which the normal force acts is **not** directly under the center of mass)

The stack will not tip over as long as the net torque is zero, hence

$$+2MaL \;-\; Mgx \;=\; 0 \quad\Rightarrow\quad x \;=\; 2\left(\frac{a}{g}\right)L \quad .$$

But this will only be possible as long as the effective point from which the normal force acts is *within* the base of the stack. So the stack will not tip over provided that

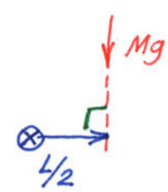
$$x \;=\; 2\left(\frac{a}{g}\right)L \;\le\; \tfrac{1}{2}L \quad\Rightarrow\quad a \;\le\; \tfrac{1}{4}g \quad .$$



From the point of view of a rider in the truck bed, the stack of cubes is stationary, but it (as well as the passenger) experiences a force directed toward the rear of the truck, in addition to the force of gravity and the normal force. The role of static frictional force here is to oppose this rearward force and hold the stack of boxes in place. Since the stack is not observed to accelerate, the equation for the horizontal forces on the stack is $f_s - Ma = 0$ and the vertical force equation is $N - Mg = 0$. We can look at the torques acting about the rearward end of the base of the stack of cubes. The static frictional force produces no torque because its direction of action is parallel to its moment arm. The other torques are:

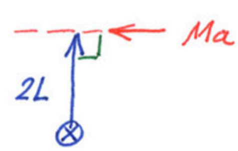
torque due to weight force – $\tau_W = \underset{(cw)}{-} \left(\frac{L}{2}\right) \cdot Mg \cdot \sin 90°$

$= -\frac{1}{2} MgL$



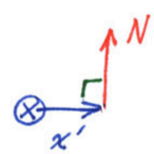
torque due to "rearward force" – $\tau_R = \underset{(ccw)}{+} 2L \cdot Ma \cdot \sin 90°$

$= +2MaL$



torque due to normal force – $\tau_N = \underset{(ccw)}{+} x \cdot N \cdot \sin 90°$

$= +(Mg) \cdot x'$



The stack will not topple over as long as the net torque is zero, so

$$+2MaL \;+\; Mgx' \;-\; \tfrac{1}{2}MgL \;=\; 0 \quad\Rightarrow\quad x' \;=\; \frac{\frac{1}{2}gL \;-\; 2aL}{g} \;=\; \left(\frac{1}{2} - \frac{2a}{g}\right)L \quad .$$

The point of effective action for the normal force must remain forward of the trailing edge of the base of the stack; thus,

$$x' \;=\; \left(\frac{1}{2} - \frac{2a}{g}\right)L \;\geq\; 0 \quad\Rightarrow\quad \underbrace{\frac{1}{2} - \frac{2a}{g} \;\geq\; 0}_{\text{since } L > 0} \quad\Rightarrow\quad a \;\leq\; \tfrac{1}{4}g \quad ,$$

as we found for the stationary reference frame.

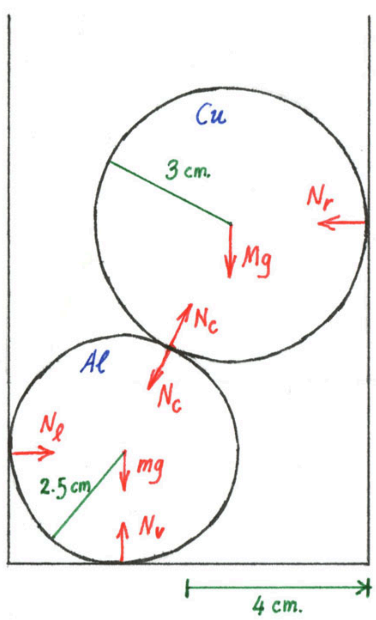
We know that the magnitude of the static friction force cannot exceed $f_{s_{\max}} \;=\; \mu_s N \;=\; \mu_s Mg \;=\; 0.32\,Mg$, so the stack will only start to slide along the truck bed if $Ma \;>\; f_{s_{\max}} \;=\; 0.32\,Mg \;\;\Rightarrow\;\; a \;>\; 0.32\,g$. Hence, the stack of cubes will topple over before it would start to slide in the truck bed.

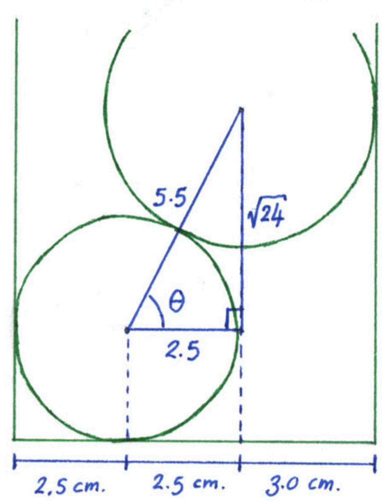
11) Since we will be comparing all of the forces to the weight of the aluminum ball, we will find it useful to find the relative masses of the copper and aluminum balls. If we call the mass of the copper ball M and that of the aluminum ball m , we have

$$\frac{M}{m} \;=\; \frac{\rho_{Cu}V_{Cu}}{\rho_{Al}V_{Al}} \;=\; \frac{\rho_{Cu}\cdot\frac{4\pi}{3}r_{Cu}^3}{\rho_{Al}\cdot\frac{4\pi}{3}r_{Al}^3} \;=\; \left(\frac{\rho_{Cu}}{\rho_{Al}}\right)\cdot\left(\frac{r_{Cu}}{r_{Al}}\right)^3 \;=\; \left(\frac{9000\;\frac{\text{kg.}}{\text{m.}^3}}{2700\;\frac{\text{kg.}}{\text{m.}^3}}\right)\cdot\left(\frac{3.0\text{ cm.}}{2.5\text{ cm.}}\right)^3$$

notice that unit conversions are unnecessary when working with comparison ratios

$$=\; \left(\frac{10}{3}\right)\cdot\left(\frac{6}{5}\right)^3 \;=\; \frac{144}{25} \;\text{ or }\; 5.76 \quad .$$





We can now analyze the forces on each ball with the aid of the two diagrams above: the one to the right provides geometrical information which will allow us to describe the angle θ above the horizontal made by the line connecting the centers of the spheres. The forces on the copper ball are

horizontal: $N_c \cos\theta - N_r = 0$; vertical: $N_c \sin\theta - Mg = 0$,

while the forces on the aluminum ball are

horizontal: $N_l - N_c \cos\theta = 0$; vertical: $N_v - mg - N_c \sin\theta = 0$,

with mg being the weight of the aluminum ball. From these equations, we find that

$$N_r = N_c \cos\theta = N_l \quad , \quad N_c \sin\theta = Mg \quad , \quad N_v = mg + N_c \sin\theta \quad .$$

We are now able to calculate the magnitudes of all the forces:

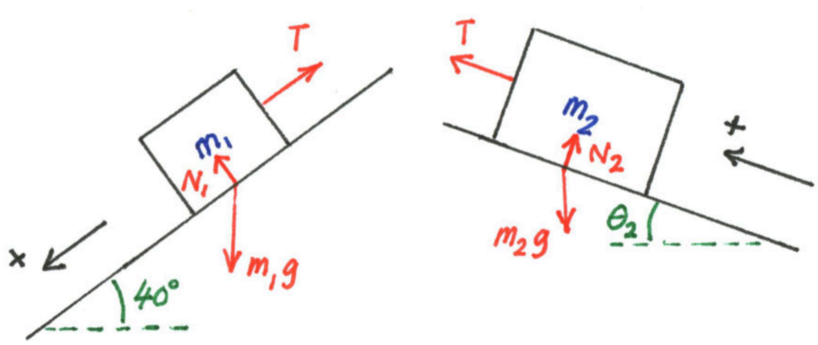
$$N_v = mg + N_c \sin\theta = mg + Mg = mg + \frac{144}{25} mg = \frac{169}{25} mg = 6.76\, mg \quad ;$$

$$N_c = \frac{Mg}{\sin\theta} = \frac{\frac{144}{25} mg}{(\sqrt{24}/5.5)} \approx 6.47\, mg \quad ; \quad \text{and}$$

$$N_r = N_l = N_c \cos\theta = \left(\frac{Mg}{\sin\theta}\right)\cos\theta = Mg \cot\theta = \left(\frac{144}{25} mg\right)\left(\frac{2.5}{\sqrt{24}}\right) \approx 2.94\, mg \quad .$$

12)

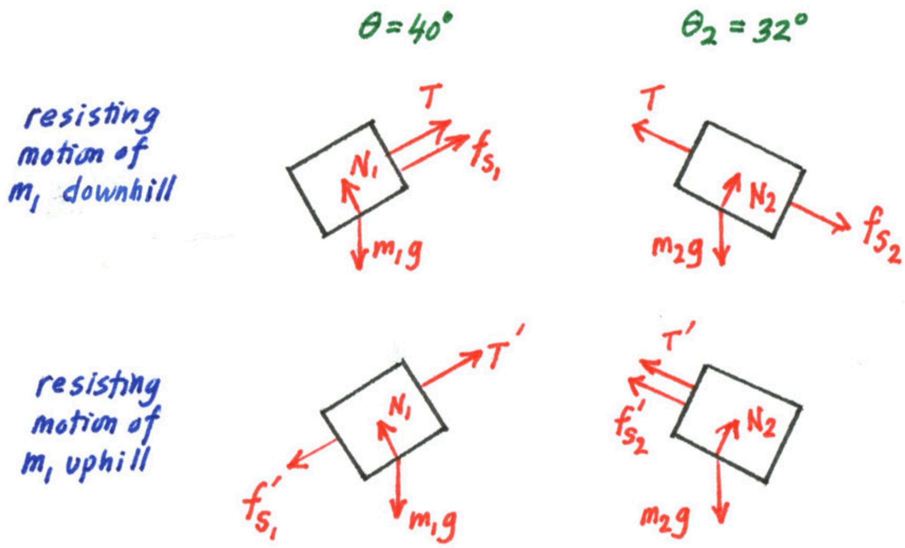
a) In this situation, due to the absence of friction, the two masses are in a marginally stable static equilibrium.



We will call the downhill motion of m_1 positive, which automatically makes the uphill motion of m_2 positive in turn. The net force on m_1 acting parallel to the incline it is on is $m_1 g \sin 40^o - T = 0$, while the net force on m_2 acting parallel to its incline is $T - m_2 g \sin\theta_2 = 0$. When we solve each of these equations for the tension T in the connecting cord and then equate the results, we find $m_1 g \sin 40^o = T = m_2 g \sin\theta_2$. Since both of the masses of the blocks are known, we can solve for the angle θ_2 that is required in order for this static equilibrium to exist:

$$\sin\theta_2 \;=\; \left(\frac{m_1}{m_2}\right)\sin 40^\circ \;\approx\; \left(\frac{1.8\ \text{kg.}}{3.0\ \text{kg.}}\right)\cdot 0.643 \;\approx\; 0.386 \quad\Rightarrow\quad \theta_2 \;\approx\; 22.7^\circ \quad .$$

b) With the inclusion of friction, the static equilibrium of the two masses on their inclines becomes far more stable. In this portion of the Problem, the angle for the incline m_2 rests upon is chosen to be $\theta_2 = 32^o$. There are now two cases to consider for the limits of static equilibrium: one in which the mass m_1 is *just* prevented from sliding downhill, the other in which it is *just* prevented from being drawn uphill.



The forces on each mass in the case where m_1 resists sliding downhill are

	m_1	m_2
forces perpendicular to incline	$N_1 - m_1 g\cos 40^\circ = m_1 a_\perp = 0$	$N_2 - m_2 g\cos 32^\circ = m_2 a_\perp = 0$
forces parallel to incline	$m_1 g\sin 40^\circ - T - f_{s_{1\max}} = m_1 a_\parallel = 0$	$T - m_2 g\sin 32^\circ - f_{s_{2\max}} = m_2 a_\parallel = 0$

From the equations for the perpendicular (normal) forces on the blocks, we obtain $N_1 = m_1 g\cos 40^o$ and $N_2 = m_2 g\cos 32^o$. Since the limits of static friction on each block are $f_{s_{1\max}} = \mu_s N_1$ and $f_{s_{2\max}} = \mu_s N_2$, the equations for the forces parallel to each incline can be solved for the tension T in the cord, yielding

$$m_1 g\sin 40^o \;-\; \mu_s m_1 g\cos 40^o \;=\; T \;=\; m_2 g\sin 32^o \;+\; \mu_s m_2 g\cos 32^o \quad .$$

If we now solve this last equation for m_2 , we have

$$m_2 \;=\; \left(\frac{\sin 40^\circ - \mu_s\cos 40^\circ}{\sin 32^\circ + \mu_s\cos 32^\circ}\right)\cdot m_1 \;\approx\; \left(\frac{0.643 \;-\; 0.38\cdot 0.766}{0.530 \;+\; 0.38\cdot 0.848}\right)\cdot 1.8\ \text{kg.} \;\approx\; 0.74\ \text{kg.} \quad ,$$

which is the smallest mass m_2 may have in order to keep m_1 from sliding downhill.

If the block m_2 is massive enough, it will instead apply enough tension in the cord to pull block m_1 uphill. The limit of static friction on m_1 in this direction is found by changing the equations for the forces parallel to each incline to

$$m_1 g \sin 40^\circ - T' \underset{\uparrow}{+} f_{s_{1\max}} = m_1 a_{\parallel} = 0 \quad \text{and} \quad T' - m_2 g \sin 32^\circ \underset{\uparrow}{+} f_{s_{2\max}} = m_2 a_{\parallel} = 0 \ ;$$

the equations for the normal forces are not affected. Repeating the rest of the calculations as we did before, we now have

$$m_2 = \left(\frac{\sin 40^\circ + \mu_s \cos 40^\circ}{\sin 32^\circ - \mu_s \cos 32^\circ} \right) \cdot m_1 \approx \left(\frac{0.643 + 0.38 \cdot 0.766}{0.530 - 0.38 \cdot 0.848} \right) \cdot 1.8 \text{ kg.} \approx 8.09 \text{ kg.} \quad ,$$

the greatest mass this block may have without overcoming static friction and pulling the other block along after it.

c) We continue to use the Atwood machine described in part (b), but now with a mass $m_2 = 9.2$ kg. We have seen that this will be more than sufficient mass to overcome static friction and set both blocks sliding along their inclines, with m_1 moving uphill. The normal forces acting on the blocks will still be as they were in part (b). However, with the blocks in motion, their accelerations will no longer be zero; with kinetic friction acting on the blocks, the equations for the forces parallel to the inclines become

$$m_1 g \sin 40^\circ - T'' + f_{k_1} = m_1 a_{\parallel} \quad \text{and} \quad T'' - m_2 g \sin 32^\circ + f_{k_2} = m_2 a_{\parallel} \quad ,$$

with $f_k = \mu_k N$. We now solve each equation for the new cord tension T'' and equate the results to find

$$m_1 g \sin 40^\circ + \mu_k m_1 g \cos 40^\circ - m_1 a_{\parallel} = T'' = m_2 g \sin 32^\circ - \mu_k m_2 g \cos 32^\circ + m_2 a_{\parallel}$$

$$\Rightarrow \quad (m_1 + m_2) \cdot a_{\parallel} = m_1 g \sin 40^\circ + \mu_k m_1 g \cos 40^\circ - m_2 g \sin 32^\circ + \mu_k m_2 g \cos 32^\circ$$

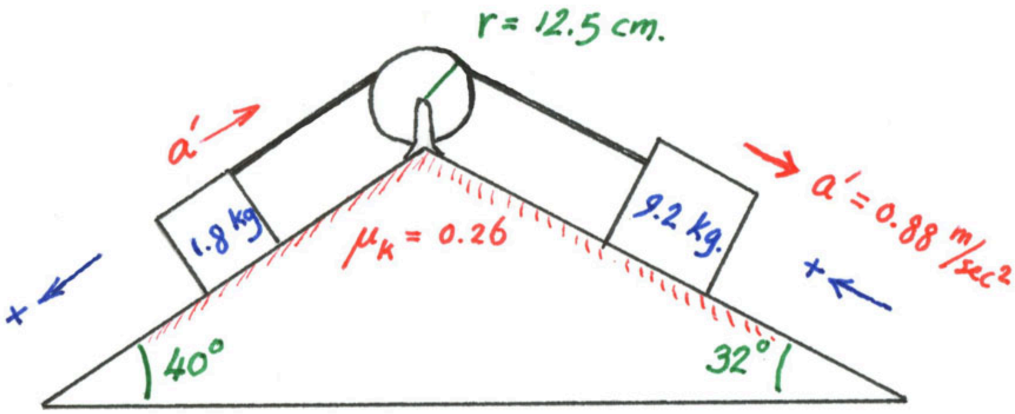
$$\Rightarrow \quad a_{\parallel} = \left[\frac{m_1 (\sin 40^\circ + \mu_k \cos 40^\circ) - m_2 (\sin 32^\circ - \mu_k \cos 32^\circ)}{m_1 + m_2} \right] \cdot g$$

$$\approx \left[\frac{1.8 \text{ kg.} \cdot (0.643 + 0.26 \cdot 0.766) - 9.2 \text{ kg.} \cdot (0.530 - 0.26 \cdot 0.848)}{1.8 + 9.2 \text{ kg.}} \right] \cdot (9.81 \frac{\text{m.}}{\text{sec.}^2})$$

$$\approx -0.121\, g \quad \text{or} \quad -1.19 \frac{\text{m.}}{\text{sec.}^2} \quad .$$

The negative sign for this acceleration agrees with our expectation that m_1 is being pulled uphill and it is m_2 that is sliding downhill.

d) With the pulley now experiencing friction with the cord connecting the blocks, the tensions in the cord on either side of the pulley are *no longer equal.*



The equations for the forces parallel to the inclines that we used in part (c) must be modified to read

$$m_1 g \sin 40^\circ - T_1 + \mu_k m_1 g \cos 40^\circ = m_1 a_{\parallel}' \quad \text{and} \quad T_2 - m_2 g \sin 32^\circ + \mu_k m_2 g \cos 32^\circ = m_2 a_{\parallel}' \; .$$

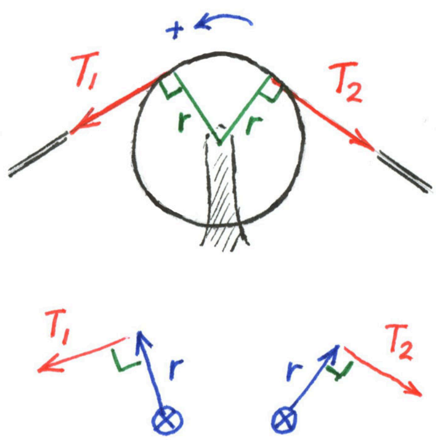
Since block m_2 is observed to be sliding downhill, the acceleration of both blocks is $a_{\parallel}' = -0.88 \frac{\text{m.}}{\text{sec.}^2}$. We can solve each of these force equations for the tensions, since they are the only unknowns, to determine

$$T_1 = m_1 \cdot \left[g(\sin 40^\circ + \mu_k \cos 40^\circ) - a_{\parallel}' \right]$$

$$\approx (1.8 \text{ kg.}) \cdot \left[(9.81 \frac{\text{m.}}{\text{sec.}^2}) \cdot (0.643 + 0.26 \cdot 0.766) - (-0.88 \frac{\text{m.}}{\text{sec.}^2}) \right] \approx 16.45 \text{ N.} \quad \text{and}$$

$$T_2 = m_2 \cdot \left[g(\sin 32^\circ - \mu_k \cos 32^\circ) + a_{\parallel}' \right]$$

$$\approx (9.2 \text{ kg.}) \cdot \left[(9.81 \frac{\text{m.}}{\text{sec.}^2}) \cdot (0.530 - 0.26 \cdot 0.848) + (-0.88 \frac{\text{m.}}{\text{sec.}^2}) \right] \approx 19.84 \text{ N.}$$



These unequal tensions apply a net torque to the pulley, so it *will* rotate. The torques about the center-of-mass of the pulley are

$$\tau_{T_1} = +r \cdot T_1 \cdot \sin 90^\circ = +T_1 r \quad \text{and} \quad \tau_{T_2} = -r \cdot T_2 \cdot \sin 90^\circ = -T_2 r \quad ,$$

so the net torque on the pulley is $(T_1 - T_2) \cdot r \approx (16.45 - 19.84 \text{ N}) \cdot 0.125$ m. ≈ -0.424 N–m. (the negative sign indicates that the rotation is clockwise, as we would expect from the diagrams).

For a rigidly rotating object (all points rotate about the axis at the same angular speed) like the pulley, the net torque is $\tau_{\text{net}} = I_{\text{CM}}\,\alpha$, where α is the angular acceleration. Since the edge of the pulley accelerates along with the cord, we have the same relationship as we do for a rolling object, $\alpha = \frac{a'}{r}$. The moment of inertia of the pulley about its center of mass is thus given by

$$I_{CM} = \frac{\tau_{net}}{\alpha} = \frac{\tau_{net}}{a'/r} = \frac{r \cdot \tau_{net}}{a'} \approx \frac{0.125 \text{ m.} \cdot (-0.424 \text{ N–m.})}{-0.88 \frac{\text{m.}}{\text{sec.}^2}} \approx 0.060 \text{ kg.– m.}^2$$

13)

a) This situation can be viewed from either of two reference frames; the Problem is solved in much the same way in both cases. From the point of view of someone standing near the conveyor belt, the graphite block falls straight down, lands with zero horizontal velocity, and is accelerated up to the speed of the belt. Fron a viewpoint *on* the belt, the block falls along a parabolic arc and lands with a horizontal speed of 3 m./sec. , after which the block is brought to rest by kinetic friction. In either reference frame, we may apply the work-kinetic energy theorem: the magnitude of the observed change in the kinetic energy of the block is equal to the magnitude of the work done on it by kinetic friction. Since the conveyor belt is horizontal, the normal force on the block is $N = Mg$, so the kinetic frictional force on the block is $f_k = \mu_k N = \mu_k Mg$. The friction acts in the opposite direction to the path along which the block slides, so the frictional work done on the block is $W_f = \vec{f}_k \bullet \Delta\vec{x} = \mu_k Mg \cdot L \cdot \cos 180^\circ = -\mu_k MgL$, where L is the distance the block slides along the belt. By the work-kinetic energy theorem then, we find

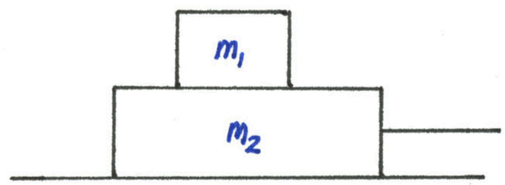
$$W_f = \Delta K \quad \Rightarrow \quad -\mu_k MgL = -\tfrac{1}{2} Mv^2$$

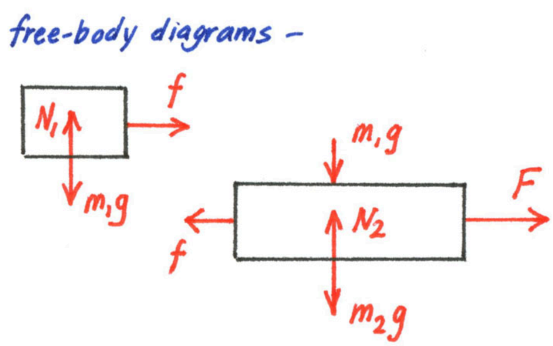
$$\Rightarrow \quad L = \frac{v^2}{2\mu_k g} \approx \frac{(3.0 \frac{\text{m.}}{\text{sec.}})^2}{2 \cdot 0.2 \cdot 9.81 \frac{\text{m.}}{\text{sec.}^2}} \approx 2.29 \text{ m.}$$

b) Here, there is no friction between any of the surfaces in contact, so we do not need to be concerned with the normal forces on the blocks. Since the only horizontal force on m_1 would be due to friction, in this case the upper block is not accelerated; thus, $a_1 = 0$. In the absence of friction, the only horizontal force acting on m_2 is the applied force $F = 6$ N. ; hence, the acceleration of the lower block is

$a_2 = \dfrac{F}{m_2} = \dfrac{6\text{ N.}}{1.3\text{ kg.}} \approx 4.62\,\dfrac{\text{m.}}{\text{sec.}^2}$. The lower block will ultimately be pulled out from under the upper one.

c) Since the contact surfaces between the blocks and between the lower block and the tabletop are horizontal, the normal forces on the blocks are simply $N_1 = m_1 g$ and $N_2 = (m_1 + m_2)\,g$. The static frictional force acting between the blocks is then $f_s \le \mu_s N_1 = \mu_s m_1 g$. So the two blocks will move together as a unit provided that this limit applies. (Since the tabletop is frictionless, N_2 is not important to know and contact with the upper block is the only source of friction.)





The horizontal force equation for the lower block is $F - f_s = m_2 a_2$, while that for the upper block is $f_s = m_1 a_1$. If the two blocks are to stay together, their accelerations must match, implying that $a_1 = \dfrac{f_s}{m_1} = a_2 = \dfrac{F - f_s}{m_2}$. When we apply the maximum possible value for the static frictional force $f_{s_{max}} = \mu_s N_1$, we find that the limit for the applied force is given by

$$\frac{\mu_s m_1 g}{m_1} = \frac{F_{max} - \mu_s m_1 g}{m_2} \quad \Rightarrow \quad F_{max} = \mu_s \cdot (m_1 + m_2) \cdot g$$

$$\approx 0.45 \cdot (0.7 + 1.3\text{ kg.}) \cdot (9.81\,\frac{\text{m.}}{\text{sec.}^2}) \approx 8.83\text{ N.}$$

Since $F_{max} > 6$ N. , a horizontal applied force at this level still permits the blocks to stay together. Because they move as a unit, their mutual acceleration is $a = \dfrac{F}{m_1 + m_2} = \dfrac{6\text{ N.}}{(0.7 + 1.3\text{ kg.})} \approx 3.00\,\dfrac{\text{m.}}{\text{sec.}^2}$. Note that this means that the static frictional force here is $f_s = m_1 a = (0.7\text{ kg.})(3.00\text{ m./sec.}^2) = 2.10$ N. , which is

certainly less than $f_{s_{max}} = \mu_s m_1 g = 0.45 \cdot (0.7 \text{ kg.}) \cdot (9.81 \frac{\text{m.}}{\text{sec.}^2}) \approx 3.09 \text{ N.}$ As a check, we find that $a_2 = \frac{F - f_s}{m_2} \approx \frac{6.00 - 2.10 \text{ N.}}{1.3 \text{ kg.}} \approx 3.00 \frac{\text{m.}}{\text{sec.}^2} = a$.

For an applied horizontal force $F = 10 \text{ N.} > F_{max}$, the blocks will *not* move together and the kinetic frictional force $f_k = \mu_k m_1 g$ must be used in our analysis. Upon replacing f_s with f_k in the horizontal force equations for each block, we now have

$$a_1' = \frac{f_k}{m_1} = \frac{\mu_k m_1 g}{m_1} = \mu_k g \approx 0.32 \cdot (9.81 \frac{\text{m.}}{\text{sec.}^2}) \approx 3.14 \frac{\text{m.}}{\text{sec.}^2} \quad \text{and}$$

$$a_2' = \frac{F - f_k}{m_2} \approx \frac{10.0 \text{ N.} - 0.32 \cdot (0.7 \text{ kg.}) \cdot (9.81 \frac{\text{m.}}{\text{sec.}^2})}{1.3 \text{ kg.}}$$

$$\approx \frac{10.0 - 2.20 \text{ N.}}{1.3 \text{ kg.}} \approx 6.00 \frac{\text{m.}}{\text{sec.}^2} \quad .$$

The lower block will be pulled out from under the upper one, but here the upper block will move forward a bit first, which was *not* the case in the situation we treated in part (a) above.

d) Nothing else about this system is changed if we apply the force to the lower block with a spring, instead of a cord. In order for the two blocks to continue moving together, the force applied must not exceed $F_{max} \approx 8.83$ N. Hooke's Law tells us that the restoring force of a spring is given by $F_{sp} = k \cdot \Delta x$, where Δx is the displacement of the spring from equilibrium. A maximum allowed force of 8.83 N. requires that the maximum permitted displacement for this spring be $(\Delta x)_{max} = \frac{F_{max}}{k} \approx \frac{8.83 \text{ N.}}{72 \text{ N./m.}} \approx 0.123 \text{ m.}$ This will set the limit on the amplitude for the pair of blocks oscillating at the end of this spring.

e) There are two phases to the motion of any of the pieces of tableware: an object is first carried along on the rapidly accelerating tablecloth by kinetic friction, then, once the fabric has been extracted from underneath it, the same object decelerates to rest due to the kinetic friction between the object and the tabletop.

We will follow the motion of one such object, which is initially at rest on the tablecloth. If that tablecloth is now pulled with an acceleration greater than $\mu_s g$, it will overcome the static friction between the object and itself, making it possible to pull the tablecloth out from under the object. However, while the object remains in contact with the tablecloth, it will be accelerated by *kinetic* friction at the rate $\mu_k g$. If we call the length of time the object rides the tablecloth t_c, then the object attains a peak speed of $v_{max} = \mu_k g t_c$ and is pulled along over the table for a distance $x_1 = ½ \mu_k g \, t_c^2$ before the tablecloth comes out from under it.

With the object now sliding across the bare tabletop, kinetic friction will act to bring it to a stop. With a coefficient of kinetic friction μ_k' between the object and the

tabletop, the acceleration of the object is now $-\mu_k' g$. The object will come to rest in an interval Δt given by

$$v_f = 0 = v_i + a\Delta t = v_{\max} + (-\mu_k' g)\Delta t = \mu_k g t_c - \mu_k' g \Delta t$$

$$\Rightarrow \quad \mu_k g t_c = \mu_k' g \Delta t \quad \Rightarrow \quad \Delta t = \left(\frac{\mu_k}{\mu_k'}\right) t_c \quad .$$

In that interval, the object will continue to slide, before finally stopping, by a distance

$$x_2 = v_{\max} \cdot \Delta t + \frac{1}{2}(-\mu_k' g) \cdot (\Delta t)^2$$

$$= (\mu_k g t_c) \cdot \left(\frac{\mu_k}{\mu_k'}\right) t_c - \frac{1}{2} \mu_k' g \left[\left(\frac{\mu_k}{\mu_k'}\right) t_c\right]^2 = \frac{\mu_k^2 \, g \, t_c^2}{2\,\mu_k'} \quad .$$

We can also find this by applying the "velocity-squared" formula:

$$v_f^2 = v_i^2 + 2a(\Delta x) \quad \Rightarrow \quad 0 = (\mu_k g t_c)^2 + 2 \cdot (-\mu_k' g) \cdot x_2 \quad \Rightarrow \quad x_2 = \frac{\mu_k^2 \, g \, t_c^2}{2\,\mu_k'} \quad .$$

We want to ensure that no object is caused to move more than 8 cm., so we require that $x_1 + x_2 \le 0.08$ m.

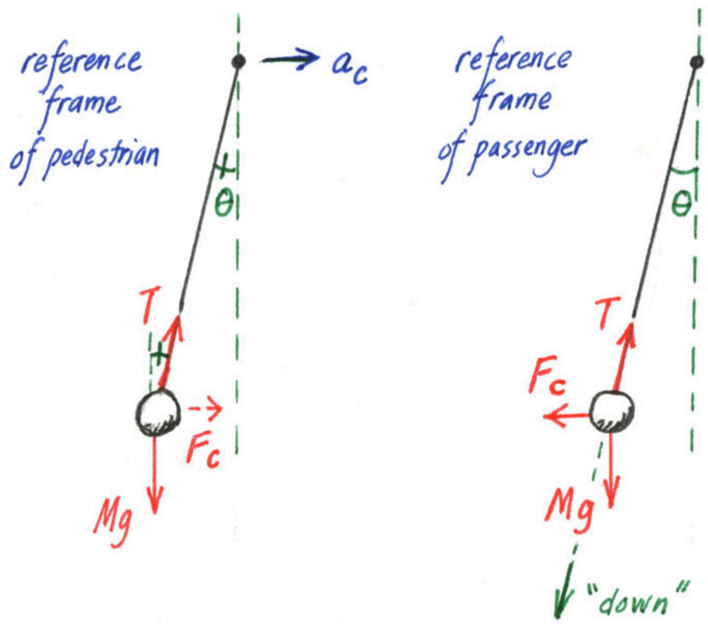
$$\Rightarrow \quad \frac{1}{2} \mu_k \, g \, t_c^2 + \frac{1}{2} \frac{\mu_k^2}{\mu_k'} g \, t_c^2 = \frac{1}{2} \left(\frac{\mu_k' + \mu_k}{\mu_k'}\right) \cdot \mu_k \, g \, t_c^2 \le 0.08 \text{ m.}$$

$$\Rightarrow \quad t_c^2 \le \frac{2 \cdot 0.08 \text{ m.}}{\mu_k \, g} \cdot \left(\frac{\mu_k'}{\mu_k' + \mu_k}\right) \approx \frac{2 \cdot 0.08 \text{ m.}}{0.15 \cdot 9.81 \frac{\text{m.}}{\text{sec.}^2}} \cdot \left(\frac{0.24}{0.24 + 0.15}\right) \approx 0.067 \text{ sec.}^2$$

$$\Rightarrow \quad t_c \le 0.26 \text{ sec.}$$

So the tablecloth must be pulled out quickly enough that no object rides on it for longer than 0.26 second; this will plainly require the tablecloth to be accelerated well above the amount needed to overcome static friction with the objects resting on it.

14) We can examine the plumb bob in the first part of this Problem using either the stationary reference frame of, say, a pedestrian on the sidewalk watching the car round the corner, or the accelerating reference frame of a passenger inside the car. This consideration of the two differing points of view will be of use in the second part of the Problem.



In the stationary reference frame, the turning vehicle is seen undergoing a centripetal acceleration of

$$a_C \;=\; \frac{v^2}{R} \;=\; \frac{\left(11\,\frac{\text{mi.}}{\text{hr.}} \cdot 5280\,\frac{\text{ft.}}{\text{mi.}} \cdot \frac{1\text{ hr.}}{3600\text{ sec.}}\right)^2}{27\text{ ft.}} \;\approx\; \frac{\left(16.1\,\frac{\text{ft.}}{\text{sec.}}\right)^2}{27\text{ ft.}} \;\approx\; 9.64\,\frac{\text{ft.}}{\text{sec.}^2}$$

$$\text{or} \quad 9.64\,\frac{\text{ft.}}{\text{sec.}^2} \cdot \frac{1\text{ m.}}{3.28\text{ ft.}} \;\approx\; 2.94\,\frac{\text{m.}}{\text{sec.}^2} \quad .$$

This acceleration is provided by the centripetal force, which is a *resultant* force in this reference frame. In the vertical direction, the force equation is

$$T\,\cos\theta \;-\; Mg \;=\; Ma_V \;=\; 0 \quad\Rightarrow\quad T\,\cos\theta \;=\; Mg \quad\Rightarrow\quad T \;=\; \frac{Mg}{\cos\theta} \quad .$$

In the radial direction, we simply have $T\sin\theta \;=\; F_c$, that is, the horizontal component of the tension in the cord of the plumb line is the centripetal force that pulls the bob around the curve. From this, we find that

$$T\,\sin\theta \;=\; F_C \quad\Rightarrow\quad \left(\frac{Mg}{\cos\theta}\right)\cdot\sin\theta \;=\; Ma_C$$

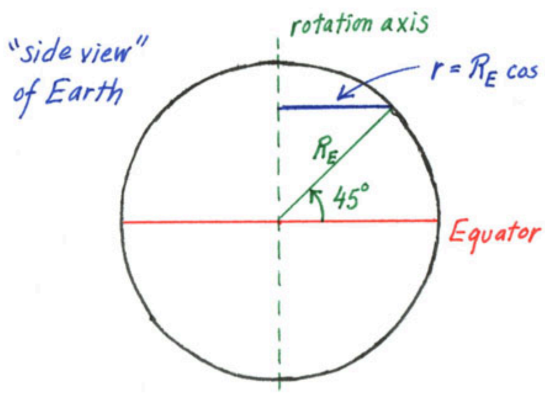
$$\Rightarrow\quad \tan\theta \;=\; \frac{a_C}{g} \;\approx\; \frac{9.64\,\frac{\text{ft.}}{\text{sec.}^2}}{32.2\,\frac{\text{ft.}}{\text{sec.}^2}} \;\left(\text{or}\;\; \frac{2.94\,\frac{\text{m.}}{\text{sec.}^2}}{9.81\,\frac{\text{m.}}{\text{sec.}^2}}\right) \;\approx\; 0.300$$

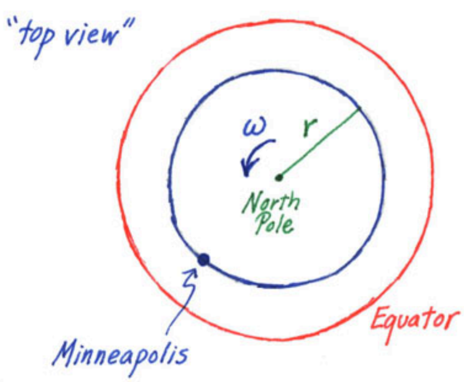
$$\Rightarrow\quad \theta \;\approx\; 16.7^\circ \quad \text{from the vertical} \quad .$$

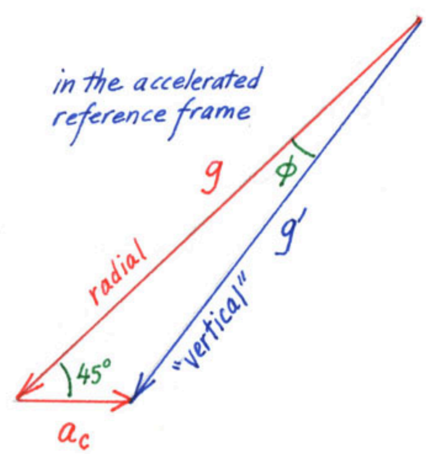
From the viewpoint of a passenger in the turning car, there is *no* acceleration of the bob; rather it is in static equilibrium under three forces: gravity (its own weight), the tension in the cord, and a centrifugal force F_C which pulls the bob to the outside of the turn. The radial force equation in this reference frame becomes $T \sin\theta - F_C = Ma_r = 0$, which then leads to the same result for θ. The Principle of Equivalence, discussed in Problem 5, can be brought in to describe this physical situation more simply: since there is no acceleration of the bob observed, it *is* hanging "straight down" in a gravitational field which happens to be tilted 16.7º with respect to a "vertical" direction defined by the body of the car. Objects will fall toward the "outside of the turn" simply because *this* gravitational field pulls them that way. (We would also find that the acceleration due to "gravity" in this tilted field is $g' = \sqrt{9.81^2 + 2.94^2} \approx 10.24 \frac{\text{m.}}{\text{sec.}^2}$.)

We can now apply some of this reasoning to the second part of this Problem. From a stationary reference frame, the Earth is seen to rotate with a period of one *sidereal* day, which is 86,164 seconds (using the mean solar day of 86,400 seconds will only change our results in the third decimal place), giving an angular speed for the surface of the Earth of $\omega = \frac{2\pi}{T} \approx \frac{2\pi \text{ rad.}}{86164 \text{ sec.}} \approx 7.29 \cdot 10^{-5} \frac{\text{rad.}}{\text{sec.}}$. So the plumb bob (and Minneapolis) are observed to travel on a circle about the Earth's rotation axis of radius $r = R_E \cos 45º$ (where R_E is the Earth's equatorial radius) with a centripetal acceleration of

$$a_C = \omega^2 r = (7.29 \cdot 10^{-5} \frac{\text{rad.}}{\text{sec.}})^2 \cdot (6378 \text{ km.} \cdot 1000 \frac{\text{m.}}{\text{km.}}) \cdot \frac{\sqrt{2}}{2} \approx 0.0240 \frac{\text{m.}}{\text{sec.}^2} .$$







Now we consider the view of an observer in accelerating Minneapolis looking at the plumb bob. The plumb line hangs "vertically" in a direction which is the vector sum of Earth's radial gravitational field (magnitude $g = 9.81$ m./sec.^{2} making an angle 45º to the Equator; this takes the Earth to be a perfect sphere) and a "centrifugal" acceleration (magnitude a_C in a direction parallel to the Equator). We can apply the Law of Cosines to the indicated vector triangle to find

$$g'^{\,2} \;=\; g^2 + a_C^2 \;-\; 2 \cdot g \cdot a_C \cdot \cos 45^\circ \;\approx\; 9.81^2 + 0.0240^2 \;-\; 2\,(9.81)\,(0.0240)\,(\frac{\sqrt{2}}{2})$$

$$\approx\; 95.90\,\frac{\text{m.}^2}{\text{sec.}^4} \quad\Rightarrow\quad g' \;\approx\; 9.793\,\frac{\text{m.}}{\text{sec.}^2} \quad .$$

The Law of Sines then gives us

$$\frac{\sin 45^\circ}{g'} \;=\; \frac{\sin\phi}{a_C} \quad\Rightarrow\quad \sin\phi \;=\; \frac{a_C}{g'} \cdot \sin 45^\circ \;\approx\; \frac{0.0240\,\frac{\text{m.}}{\text{sec.}^2}}{9.793\,\frac{\text{m.}}{\text{sec.}^2}} \cdot (\frac{\sqrt{2}}{2}) \;\approx\; 0.00173$$

$$\Rightarrow\quad \phi \;\approx\; 0.099^\circ \quad ,$$

which is the deviation of the local "vertical" direction in Minneapolis from the radial direction (the acceleration due to apparent gravity is also a little bit less than it would be for a non-rotating Earth). To put it another way, "down" does not point exactly toward the Earth's center, but rather at a place about 10 km. closer to the South Pole along the rotation axis. It is these slight deviations in the effective gravitational field of the spinning Earth that gives our planet a slightly flattened shape (called an oblate spheroid), with the polar radius being about 21 km. (0.34%) smaller than the equatorial radius.

15)

a) Without the action of air drag, the fall of a raindrop would simply obey conservation of mechanical energy. If we assume that it leaves the cloud at an altitude of 3000 meters starting nearly at rest, then the raindrop would reach the ground at a speed given by

$$K_i \;+\; U_i \;=\; K_f \;+\; U_f \quad\Rightarrow\quad \tfrac{1}{2}\, m \cdot 0^2 \;+\; mgh_i \;=\; \tfrac{1}{2}\, m\, v_f^2 \;+\; mg \cdot 0$$

$$\Rightarrow\quad v_f^2 \;=\; 2gh_i \;\approx\; 2\,(\,9.81\text{ m./sec.}^2\,)\,(\,3000\text{ m.}\,) \;\approx\; 59{,}000\ \text{m.}^2/\text{sec.}^2$$

g is pretty nearly constant
over that height (see Problem 3)

$$\Rightarrow\quad v_f \;\approx\; 240\text{ m./sec. (or } 540\text{ mi./hr.)} \quad .$$

When air resistance is taken into account, the raindrop will cease to accelerate when the upward drag force becomes equal to the downward weight force acting on the drop. If we assume that the drag is proportional to cross-sectional area and to the square of the drop's velocity, and that the droplet retains its spherical shape while falling (this last assumption is not actually realistic), then the drag force is modeled by the equation $F_D \;=\; kAv^2$. The falling droplet will then reach a "terminal velocity" when the drag and weight forces balance, given by

$$F_D - mg = ma = 0 \quad \Rightarrow \quad kAv_t^2 = mg \quad \Rightarrow \quad k \cdot \pi r^2 \cdot v_t^2 = \rho \cdot \frac{4\pi}{3} r^3 \cdot g$$

we have used the area of a circle for the cross-section of the spherical drop, $m = \rho V$ for the mass of the drop, and the formula for the volume of a sphere

$$\Rightarrow \quad v_t^2 = \frac{4g\rho r}{3k} \quad ,$$

where ρ is the density of water and k is the (assumed constant) drag coefficient.

We can solve this to express the radius of the droplet in terms of its terminal velocity thus: $r = \dfrac{3kv_t^2}{4g\rho}$. If we now compare the slower-falling raindrop with the faster one, we find

$$\frac{r'}{r} = \frac{\left(\frac{3k}{4g\rho}\right) v'^2_t}{\left(\frac{3k}{4g\rho}\right) v_t^2} = \left(\frac{v_t'}{v_t}\right)^2 = \left(\frac{1 \frac{\text{m.}}{\text{sec.}}}{5 \frac{\text{m.}}{\text{sec.}}}\right)^2 = \frac{1}{25} \quad .$$

As for the formation of the drop of water within the raincloud, if its surface area grows at a constant rate, then the drop's surface area will be proportional to the time spent in the cloud; thus, $S = 4\pi r^2 = Ct$. Upon comparing the amounts of time required for each drop to form, according to this model, we have

$$\frac{S'}{S} = \frac{4\pi r'^2}{4\pi r^2} = \frac{Ct'}{Ct} \quad \Rightarrow \quad \left(\frac{r'}{r}\right)^2 = \left(\frac{1}{25}\right)^2 = \frac{1}{625} = \frac{t'}{t} \quad .$$

Thus, a raindrop which lands at 5 m./sec. is predicted to be 25 times larger in diameter than one which lands at 1 m./sec. and requires 625 times longer to grow to that size before falling out of its cloud. This suggests that conditions within rainclouds must differ considerably to produce different sizes of raindrops.

b) Hailstones will remain aloft as long as the air drag from the violent winds within a storm cloud can overcome their weight. If we make the same assumptions regarding them that we did for raindrops (spherical shapes, constant densities, and constant drag coefficients regardless of size), we can use the result we obtained earlier, $v_t^2 = \dfrac{4g\rho r}{3k}$, and apply this to the wind speed relative to an individual hailstone.

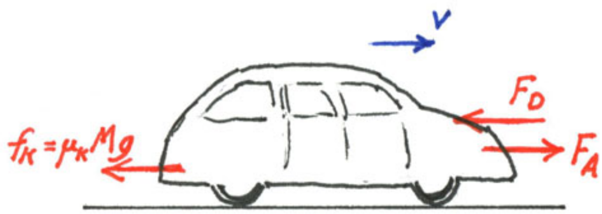
If we compare the wind speed required to support baseball-sized hail to that needed to pea-sized hail aloft, we can estimate that

$$\frac{v'^2_t}{v_t^2} = \frac{\left(\frac{4g\rho}{3k}\right) r'}{\left(\frac{4g\rho}{3k}\right) r} = \frac{r'}{r} = \frac{2r'}{2r} = \frac{7.5 \text{ cm.}}{0.75 \text{ cm.}} = 10 \quad \Rightarrow \quad \frac{v_t'}{v_t} = \sqrt{10} \approx 3.2 \quad .$$

So the turbulent wind speeds in a storm cloud need to be about three times faster to support the largest sorts of hailstones than are required for the more typical sizes of hail. (Winds within the most violent cumulonimbus clouds can exceed 120 mph.)

16)

a) The three horizontal forces acting on the car are the applied force from the engine, $\vec{F}_A$, the kinetic frictional force between the tires and the road, $\vec{f}_k$, and the drag force from the air, $\vec{F}_D$. Since the vehicle is on a level road here, the normal force on it from the road surface is $N = Mg$, so the kinetic frictional force is $f_k = \mu_k Mg$.



With the car traveling at *constant* speed, the net horizontal force on the car is $\vec{F}_{net_x} = \vec{F}_A + \vec{f}_k + \vec{F}_D = 0 \Rightarrow F_{net_x} = F_A - f_k - F_D = 0$. The net power applied to the car will then also be zero:

$$P_{net} = \vec{F}_{net_x} \cdot \vec{v} = \vec{F}_A \cdot \vec{v} + \vec{f}_k \cdot \vec{v} + \vec{F}_D \cdot \vec{v} = \vec{F}_A v \cos 0^\circ + \vec{f}_k v \cos 180^\circ + \vec{F}_D v \cos 180^\circ$$

$$= F_A v - f_k v - F_D v = 0 \quad .$$

We are told that the power being provided by the engine is $P_A = F_A v = 95{,}000$ W . At a speed of $v = 65 \frac{\text{mi.}}{\text{hr.}} \cdot 1609 \frac{\text{m.}}{\text{mi.}} \cdot \frac{1 \text{ hr.}}{3600 \text{ sec.}} \approx 29.05 \frac{\text{m.}}{\text{sec.}}$, the portion of this power that is being used to overcome air drag is

$$P_D = F_D v = F_A v - f_k v = P_A - \mu_k M g v$$

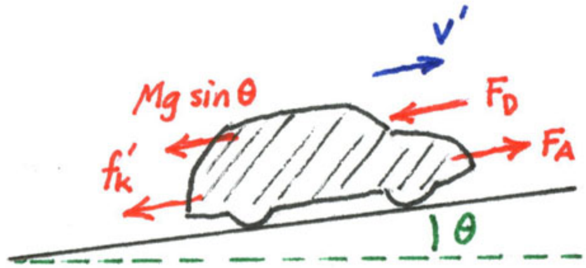
$$= 95{,}000\, W - 0.025 \cdot (900 \text{ kg.})(9.81 \frac{\text{m.}}{\text{sec.}^2})(29.05 \frac{\text{m.}}{\text{sec.}})$$

$$\approx 95{,}000 - 6410\, W \approx 88{,}600\, W \quad .$$

To maintain the car at this speed, $\frac{88{,}600\, W}{95{,}000\, W} \approx 0.933$ of the engine's power is being used to overcome air drag.

b) If the drag force is proportional to the square of the vehicle's speed, then at 80 mph, the drag is $\frac{F_D'}{F_D} = \frac{v'^2}{v^2} = \left(\frac{80 \text{ mph}}{65 \text{ mph}}\right)^2 \approx 1.515$ times stronger than it is at 65 mph. The power necessary to overcome air drag at this higher speed, however, is $\frac{P_D'}{P_D} = \frac{F_D' v'}{F_D v} = \left(\frac{v'^2}{v^2}\right) \cdot \frac{v'}{v} = \left(\frac{80 \text{ mph}}{65 \text{ mph}}\right)^3 \approx 1.864$ times greater than at 65 mph. This gives us $P_D' \approx 1.864\, P_D \approx 1.864 \cdot 88{,}600$ W $\approx 165{,}200$ W . The total power the engine must now supply is $P_A' = P_D' + P_f \approx 165{,}200 + 6410$ W $\approx 171{,}600$ W (230 hp) .

c) With the car now moving up an incline, the normal force from the road surface becomes $N' = Mg \cos\theta$, making the kinetic frictional force $f_k' = \mu_k Mg \cos\theta$. In addition to the forces already described, the weight force on the vehicle now also has a component parallel to the incline, $W_{\parallel} = Mg \sin\theta$.



The net force on the car is now $\vec{F}_{net_{\parallel}} = \vec{F}_A + \vec{F}_D + \vec{f}_k' + \vec{W}_{\parallel} = 0$, so the net power becomes $P_{net} = F_A v - F_D v - (\mu_k Mg \cos\theta)\, v - (Mg \sin\theta)\, v = 0$. Since the car is on a 12% grade, $\tan\theta = 0.12 \Rightarrow \sin\theta \approx 0.1191$, $\cos\theta \approx 0.9929$. We have seen that the power exerted by air drag is $P_D \approx 88{,}600$ W at 29.05 m./sec. (65 mph) and that it is proportional to v^3, so we can write $P_D = 88{,}600 \left(\dfrac{v}{29.05}\right)^3 W$. The power which must be delivered by the engine in order for the vehicle to maintain a speed v on the 12% upward incline is then

$$P_A = F_A v = P_D + (\mu_k Mg \cos\theta)\, v + (Mg \sin\theta)\, v$$

$$\approx 88{,}600 \left(\frac{v}{29.05}\right)^3 + (0.025)(900\ kg.)(9.81\ \frac{\text{m.}}{\text{sec.}^2})(0.9929)\, v + (900\ kg.)(9.81\ \frac{\text{m.}}{\text{sec.}^2})(0.1191)\, v$$

$$\approx 3.614\, v^3 + 219.2\, v + 1052\, v \approx 3.614\, v^3 + 1271\, v \quad .$$

We are going to keep the power from the engine at the same level it had in part (b) above, so we need to solve for v the equation $3.614\, v^3 + 1271\, v = 171{,}600$. It is not very convenient to solve this last equation directly (we can, of course, use graphing software, as will be discussed below), but we don't need to resort to trial-and-error completely either. We know that this amount of power on level ground allows the car to travel at 80 mph ($\approx$ 35.8 m./sec.), so we might expect that on a 12% climb, the car might move, say, 10% or so slower. Let's make a first guess that the solution is $v = 32$ m./sec. Since the cubic term in the equation, $3.614\, v^3$, changes much more rapidly than the linear term, $1271 v$, we'll simply set this term to $1271 \cdot 32$ for the present and solve the following for v:

$$3.614\, v^3 + 1271 \cdot 32 = 3.614\, v^3 + 40670 = 171{,}600$$

$$\Rightarrow \quad 3.614\, v^3 = 130{,}900 \quad \Rightarrow \quad v^3 \approx 36{,}200 \quad \Rightarrow \quad v \approx 33.1\ \frac{\text{m.}}{\text{sec.}} \quad ,$$

which is pretty close to our initial guess. If we adjust the linear term to accommodate this new value, we find

$$3.614\,v^3 + 1271 \cdot 33.1 \;=\; 3.614\,v^3 + 42070 \;=\; 171{,}600 \quad\Rightarrow\quad v \;\approx\; 33.0\,\frac{\text{m.}}{\text{sec.}} \quad .$$

Our solution has stabilized to one decimal place, so we may stop here. A more precise solution, using graphing software to find the x-intercepts for the function $3.614\,v^3 \;+\; 1271\,v \;-\; 171{,}600$, gives us $v \approx 32.98$ m./sec. , so our result is acceptable. As a check, we can calculate the individual power terms:

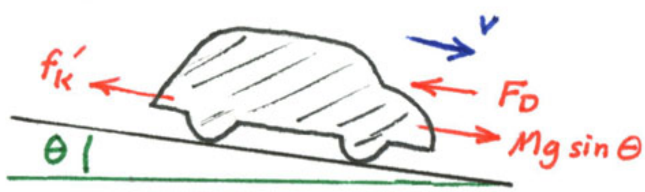
power exerted against drag:	$P_A \approx 3.614 \cdot 33.0^3 \approx 129{,}900$ W
power exerted against friction:	$P_f \approx 219.2 \cdot 33.0 \approx 7230$ W
power exerted against gravity:	$P_W \approx 1052 \cdot 33.0 \approx \underline{34{,}700 \text{ W}}$
total power to be delivered by engine:	171,800 W ,

agreeing with the intended total value to about 0.12% .

d) Without the applied force from the engine, and assuming that there is no internal friction in the drive train of the car (a bit unrealistic), the net force on the vehicle acting parallel to the downward incline is

$$\vec{F}_{net_{\parallel}}{}' \;=\; \vec{W}_{\parallel} + \vec{F}_D + \vec{f}_k{}' \;=\; 0 \quad\Rightarrow\quad F_{net_{\parallel}}{}' \;=\; W_{\parallel} - F_D - f_k{}' \;=\; 0 \quad ,$$

making the net power $P_{net} \;=\; (\,Mg \sin\theta\,)\,v \;-\; F_D v \;-\; (\,\mu_k\,Mg \cos\theta\,)\,v \;=\; 0$.

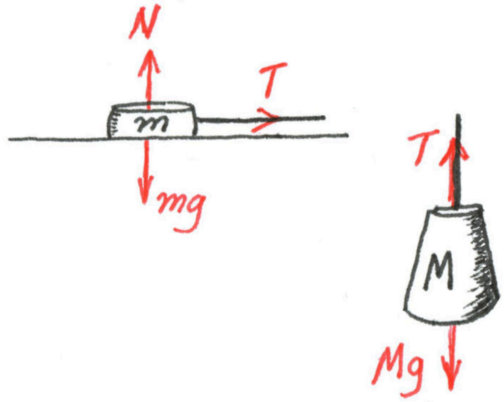


Each of these terms has the same value as it did in part (c) above, giving us

$$1052\,v \;-\; 3.614\,v^3 - \;219.2\,v \;\approx\; 832.3\,v \;-\; 3.614\,v^3 \;=\; 0 \quad ,$$

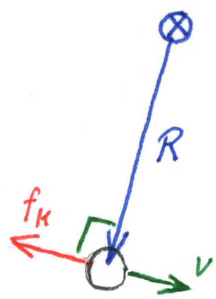
which is much easier to solve for v than the power equation we found in the previous part. We can factor this as $v \cdot (832.3 \;-\; 3.614\,v^2) \;=\; 0$, so either $v = 0$ (the uninteresting solution, since the power terms are of course all zero when the car is parked) or $832.3 \;-\; 3.614\,v^2 \;=\; 0 \;\Rightarrow\; v^2 \approx 230.3 \;\Rightarrow\; v \approx 15.2$ m./sec. (34 mph) . In a real vehicle, internal friction would make the downhill coasting speed somewhat lower than this.

17) With the puck moving on a frictionless surface, it is possible to arrange a steady-state situation (not a static, but rather a *dynamic* equilibrium) in which the puck travels on a circle at a constant tangential speed v . The centripetal force keeping the puck moving on its circle is provided by the physical force of tension in the cord connecting it to the hanging weight. If we set things up so that the suspended mass remains at rest, then the vertical force equation for this mass is $T - Mg \;=\; Ma_V \;=\; 0 \;\Rightarrow\; T \;=\; Mg$. The centripetal force on the puck is thus $F_C \;=\; \dfrac{mv^2}{R} \;=\; T \;=\; Mg$.



The tangential speed which the puck should be given in order to maintain this equilibrium is then given by

$$v^2 \;=\; \frac{M}{m}\cdot gR \;\approx\; \frac{1.3\text{ kg.}}{0.3\text{ kg.}}\,(9.81\,\frac{\text{m.}}{\text{sec.}^2})\,(0.45\text{ m.}) \;\approx\; 19.1\,\frac{\text{m.}^2}{\text{sec.}^2} \quad\Rightarrow\quad v \;\approx\; 4.37\,\frac{\text{m.}}{\text{sec.}} \quad .$$



If we now introduce a frictional force by considering the motion of the puck on a *real* air table, the kinetic friction produces a torque on the revolving puck, causing a change in its angular momentum. (*Note:* the friction will also dissipate the mechanical energy of the system, causing the hanging mass to descend and the puck to move toward the center by spiraling inward; we are not in a position in this course, however, to analyze the changes in a system where the circulating object *changes distance* from the axis of rotation – this requires somewhat more advanced mechanics.) If the rate of change is not too rapid, the puck's velocity, and hence the direction of the kinetic frictional force, will be very nearly perpendicular to the cord. The angular momentum at the moment described in the Problem is then

$$\vec{L}_0 \;=\; \vec{r}_0 \times \vec{p}_0 \quad\Rightarrow\quad L_0 \;\approx\; R_0\cdot mv_0\cdot\sin 90^\circ \;=\; R_0\cdot m\cdot\left(\frac{M}{m}\cdot gR_0\right)^{1/2} \;=\; \left(gmM\right)^{1/2}\cdot R_0^{3/2}$$

and the frictional torque is

$$\vec{\tau}_f \;=\; \vec{r}\times\vec{f}_k \quad\Rightarrow\quad \tau_f \;\approx\; R\cdot\mu_k N\cdot\sin 90^\circ \;=\; \mu_k mg\,R \quad .$$

Since this torque acts in the *opposite* direction to the angular momentum, we can write

$$-\mu_k mg\,R \;\approx\; \tau_f \;=\; \frac{dL}{dt} \;\approx\; \frac{d}{dt}\left[\left(gmM\right)^{1/2}\cdot R^{3/2}\right] \;=\; \tfrac{3}{2}\left(gmM\right)^{1/2}\cdot R^{1/2}\,\frac{dR}{dt}$$

$$\Rightarrow \quad \frac{dR}{dt} \approx \frac{-\mu_k mg\,R}{\frac{3}{2}(gmM)^{1/2}R^{1/2}} = -\underbrace{\frac{2}{3}\mu_k\left(g\,\frac{m}{M}\right)^{1/2}}_{A} R^{1/2} \quad .$$

A is a constant which
we shall refer to below

At the moment described in the Problem, we find

$$\left(\frac{dR}{dt}\right)_0 \approx -\frac{2}{3}\mu_k\left(g\,\frac{m}{M}\right)^{1/2} R_0^{1/2} \approx -\frac{2}{3}(0.0008)\left(9.81\,\frac{\text{m.}}{\text{sec.}^2}\cdot\frac{0.3\text{ kg.}}{1.3\text{ kg.}}\right)^{1/2}(0.45\text{ m.})^{1/2}$$

$$\approx -0.00054\,\frac{\text{m.}}{\text{sec.}} \quad \text{or} \quad -0.54\,\frac{\text{mm.}}{\text{sec.}}$$

Since the puck is getting closer to the hole in the table and is attached to the hanging mass by the same cord, that mass is descending at this rate as well. The spiraling-in of the puck is quite slow, so we can reasonably apply the tangential speed relationship we found earlier, $v^2 = \frac{M}{m}\cdot gR$. We see from this that the tangential speed becomes smaller as the radius at which the puck travels shrinks. So the puck is slowing down as it spirals in toward the hole.

We can examine this in somewhat more detail by using the differential equation we have determined, $\frac{dR}{dt} = -AR^{1/2}$, with $A > 0$. By rearranging this equation (in what is called a "separation of variables"), we can then integrate both sides to solve for the radius of the (approximate) circle on which the puck is moving as a function of time:

$$\frac{dR}{dt} = -AR^{1/2} \quad \Rightarrow \quad \int\frac{dR}{R^{1/2}} = -A\int dt \quad \Rightarrow \quad \frac{1}{1/2}R^{1/2} = -At + C \quad ;$$

A being the constant
shown above

since, at $t = 0$, $R(0) = R_0$, we have $2R_0^{1/2} = -A\cdot 0 + C \Rightarrow C = 2R_0^{1/2}$, and hence

$$R^{1/2} = -\frac{1}{2}\left[\frac{2}{3}\mu_k\left(g\,\frac{m}{M}\right)^{1/2}\right]\cdot t + \frac{1}{2}\cdot 2R_0^{1/2} \quad \Rightarrow \quad R(t) = \left\{R_0^{1/2} - \left[\frac{1}{3}\mu_k\left(g\,\frac{m}{M}\right)^{1/2}\right]t\right\}^2$$

and, using the tangential speed equation above, we obtain

$$v(t) = g\,\frac{M}{m}\cdot R^{1/2} = g\,\frac{M}{m}\cdot\left\{-\left[\frac{1}{3}\mu_k\left(g\,\frac{m}{M}\right)^{1/2}\right]\cdot t + R_0^{1/2}\right\}$$

$$= g\,\frac{M}{m}R_0^{1/2} - \left[\frac{1}{3}\mu_k\left(g\,\frac{m}{M}\right)^{3/2}\right]t \quad .$$

We can now estimate the time T at which the radius of the circle the puck moves along drops to zero

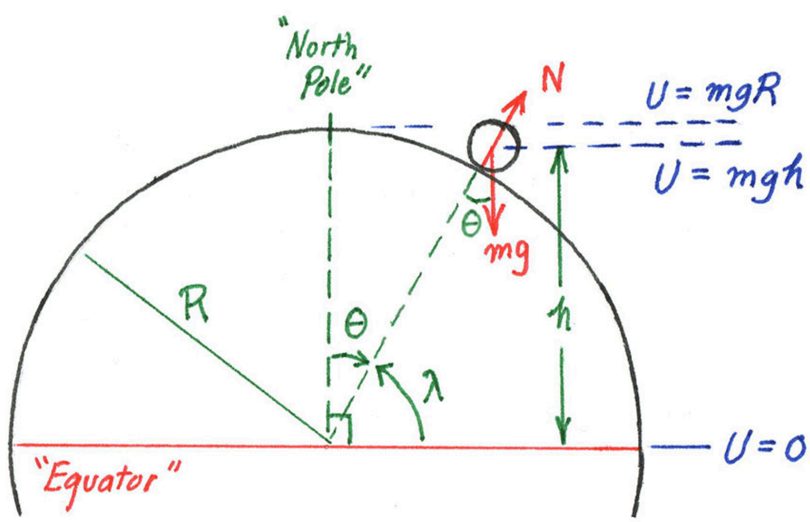
$$R_0^{1/2} - \left[\frac{1}{3}\mu_k \left(g\frac{m}{M} \right)^{1/2} \right] T = 0 \quad \Rightarrow \quad T = \frac{R_0^{1/2}}{\frac{1}{3}\mu_k \left(g\frac{m}{M} \right)^{1/2}} = \frac{3}{\mu_k}\left(\frac{M R_0}{mg} \right)^{1/2}$$

$$\approx \frac{3}{0.0008}\left(\frac{1.3\ \text{kg.} \cdot 0.45\ \text{m.}}{0.3\ \text{kg.} \cdot 9.81\ \frac{\text{m.}}{\text{sec.}^2}} \right)^{1/2} \approx 1700\ \text{sec.} \quad ;$$

we find that the tangential speed also becomes zero at this time ($v(T) = 0$) . Related physical quantities, such as the angular momentum of the puck, $L(t) = (gmM)^{1/2} R^{3/2}$, the frictional torque acting on it, $\tau(t) = -\mu_k mgR$, and the rotational kinetic energy of the puck, $K_{rot}(t) = \frac{1}{2}I\omega^2 = \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2 = \frac{1}{2}mv^2 = \frac{1}{2}m \cdot \left(\frac{M}{m}\right)gR = \frac{1}{2}MgR$, all fall smoothly to zero as the time approaches T . (It should be said that the time T is only an estimate since the approximation that the puck is traveling on diminishing circles breaks down when the radius becomes less than several centimeters.)

18) For the object rolling on the surface of the globe, there are two forces acting upon it (we can neglect rolling friction): the normal force, N , and the weight force, mg . What will be of interest to us are the components of these forces in the radial direction, as their resultant provides the centripetal force holding the object on its circular motion as it travels down a meridian of the globe's surface. If we call the radial direction toward the center of the globe positive and θ is the "polar angle" measured downward from the top of the sphere, the force in the inward radial direction is

$$mg\cos\theta - N = F_C = \frac{mv^2}{R}$$, with R being the radius of the globe.



By itself, the force equation is not of enough help to us: we also need to know something about the speed of the object as it rolls. It is not convenient to determine this from analyzing the forces, since the acceleration is not constant (or easily integrated). Since we are treating friction as negligible, mechanical energy is conserved; if we compare this total energy at the top of the sphere ($\theta = 0$, which we take to be the North Pole of the globe) and at polar angle θ , we find

$$K_i + U_i = 0 + mgR = K_f + U_f = (K_{lin} + K_{rot}) + mgh \ .$$

The linear kinetic energy of the rolling object is $K_{lin} = ½ \, mv^2$, while its rotational kinetic energy is $K_{rot} = ½ \, I \omega^2 = ½ \, (Cmr^2) \cdot (v/r)^2 = ½ \, Cmv^2$, where C is the object's coefficient of rotational inertia, and rolling requires that $\omega = v/r$, r being the radius of the object. With $h = R \cos \theta$, our conservation of energy equation becomes

$$mgR = \frac{1}{2} mv^2 + \frac{1}{2} Cmv^2 + mgR \cos\theta \quad \Rightarrow \quad v^2 = \frac{2 \cdot gR\,(1 - \cos\theta)}{1 + C} \ .$$

We now return to the force equation. The rolling object will remain on the surface of the globe as long as $N > 0$; it breaks contact with the surface at the point where the normal force drops to zero. At that position, we can say that

$$mg\cos\theta - 0 = \frac{mv^2}{R} \quad \Rightarrow \quad g\cos\theta = \frac{\left[\dfrac{2 \cdot gR\,(1 - \cos\theta)}{1 + C} \right]}{R} \quad \Rightarrow \quad \cos\theta = \frac{2\,(1 - \cos\theta)}{1 + C}$$

$$\Rightarrow \quad (1 + C)\cos\theta = 2 - 2\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{2}{3 + C} \ .$$

For a marble of uniform density, which is a solid sphere, $C = 2/5$, so the marble leaves the surface of the globe at $\cos\theta = \dfrac{2}{3 + \frac{2}{5}} = \dfrac{2}{(\frac{17}{5})} = \dfrac{10}{17} \Rightarrow \theta \approx 54.0^\circ$; since the latitude on the globe is $\lambda = 90^o - \theta$, the marble flies off the globe at $\lambda \approx 36.0^o$. The finger ring behaves nearly like an idealized hoop with $C \approx 1$, so it leaves the globe's surface where $\cos\theta = \dfrac{2}{3 + 1} = \dfrac{2}{4} = \dfrac{1}{2} \Rightarrow \theta = 60.0^\circ \Rightarrow \lambda = 30.0^\circ$.

By contrast, a sliding block will have no rotational kinetic energy, so $C = 0$, giving $\cos\theta = \dfrac{2}{3 + 0} = \dfrac{2}{3} \Rightarrow \theta = 48.2^\circ \Rightarrow \lambda = 41.8^\circ$. Since this mass does not rotate, it builds up speed on the globe's surface faster and so overcomes earlier the ability of gravity to hold it along the curved surface.

-- G.Ruffa
original notes developed during 2007-09
revised: August 2010